Geometric phase for neutrino propagation in a transverse magnetic field

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Introduction

The geometric phase [1] is a general property of quantum systems which arises if the Hamiltonian governing the system contains two or more time-dependent parameters. If the system is initially prepared in an eigenstate of the Hamiltonian then as the system undergoes an adiabatic evolution along a closed curve in the parameter space, the eigenstate develops a geometric phase in addition to the usual dynamical phase. The most common example where geometric phase arises is the spin-precession of a particle with an intrinsic magnetic moment in a magnetic field.
The minimally extended standard model (MESM) predicts the magnetic moment of a massive Dirac neutrino of mass m_ν as [2]

$$\mu_\nu = 3.2 \times 10^{-19} \left(\frac{m_\nu}{1eV}\right) \mu_B \qquad (1)$$

where μ_B is the Bohr magneton,
$\mu_B = \frac{e\hbar}{2m_e c} = 5.8 \times 10^{-15} MeV/G$
A number of models based on physics beyond MESM predict magnetic moments as large as $\sim 10^{-10}\mu_B$ for neutrinos (see [3]). Also the GEMMA experiment [4], based on antineutrino-electron scattering, puts an upper bound on neutrino magnetic moment at $2.9 \times 10^{-11}\mu_B$.
A non-zero magnetic moment causes the neutrino spin to precess in the presence of a magnetic field rotating in the transverse plane. The neutrino propagation in such a magnetic field may result in spin flip from left-handed neutrino to right-handed neutrino, thereby making it sterile. Such situations are encountered in astrophysical environments where neutrinos propagate over large distances in magnetic field in vacuum and in matter.
The possibility of geometric phase associated with neutrino spin precession in transverse magnetic field was first explored by Smirnov [5] in the context of solar neutrino problem, in which he showed that the geometric phase may induce resonant spin conversion of neutrinos inside the Sun. Here we explicitly calculate the geometric phase arising due to neutrino propagation in rotating transverse magnetic fields in matter. We limit our self only to the case of Thomas precession.

Geometric Phase

The equation describing the propagation of two helicity components of a neutrino (ν_R, ν_L) through matter in a transverse magnetic field $\mathbf{B} = B_x + iB_y = B(t)e^{i\phi(t)}$ is [5]

$$i\frac{d}{dt}\begin{pmatrix}\nu_R \\ \nu_L\end{pmatrix} = \begin{pmatrix} V/2 & \mu_\nu B e^{-i\phi} \\ \mu_\nu B e^{i\phi} & -V/2 \end{pmatrix}\begin{pmatrix}\nu_R \\ \nu_L\end{pmatrix} \qquad (2)$$

where V is the term due to neutrino mass and its interaction with matter. It is given by $V = \sqrt{2}G_F n^{eff} - \frac{\Delta m^2}{2E}$; G_F is the Fermi's constant, $\Delta m^2 = m^2(\nu_L) - m^2(\nu_R)$, E is the neutrino energy, n^{eff} is the effective concentration of particles interacting with neutrinos,

$$n^{eff} = \begin{cases} (n_e - n_n), & \text{for } \nu_{eL} - \bar{\nu}_{\mu R} \\ (n_e - n_n/2), & \text{for } \nu_{eL} - \nu_{eR} \end{cases}$$

n_e and n_n are concentrations of electrons and neutrons respectively.
The instantaneous eigenvalues and corresponding normalised eigenvectors of the Hamiltonian are

$$\lambda_\pm = \pm\sqrt{\left(\frac{V}{2}\right)^2 + (\mu_\nu B)^2} \qquad (3)$$

$$|+\rangle = \frac{1}{N}\begin{pmatrix}\mu_\nu B \\ -e^{i\phi}(\frac{V}{2} - \lambda_+)\end{pmatrix} \qquad (4)$$

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$$|-\rangle = \frac{1}{N}\begin{pmatrix} e^{-i\phi}(\frac{V}{2} - \lambda_+) \\ \mu_\nu B \end{pmatrix} \quad (5)$$

where

$$N = \sqrt{\left(\frac{V}{2} - \lambda_+\right)^2 + (\mu_\nu B)^2} \quad (6)$$

If a neutrino is initially (t=0) in a state $|+\rangle$, then as it propagates in the magnetic field it acquires a geometric phase

$$\gamma_+ = i\oint_C \mathbf{dr}\cdot\langle + | \bigtriangledown + \rangle \quad (7)$$

where the integral is over closed curve C in the parameter space. For our case, assuming cylindrical polar coordinates (B, ϕ, z) for $\mathbf{B}$, we get

$$\langle + | \bigtriangledown + \rangle = \frac{i}{N^2 B}\left(\frac{V}{2} - \lambda_+\right)^2 \hat{\phi} \quad (8)$$

The geometric phase then comes out to be

$$\gamma_+ = i\int_0^{2\pi} Bd\phi\,\langle + | \bigtriangledown + \rangle = -\frac{2\pi}{N^2}\left(\frac{V}{2} - \lambda_+\right)^2 \quad (9)$$

Since geometric phase is ambiguous upto an addition of 2π, we can write

$$\gamma_g = \gamma_+ + 2\pi = 2\pi\left[1 - \frac{1}{1 + \left(\frac{1}{\xi - \sqrt{\xi^2+1}}\right)^2}\right] \quad (10)$$

where $\xi = \frac{V}{2\mu_\nu B}$.

Results and Discussion

We can estimate the asymptotic behaviour of geometric phase with respect to magnetic field from (10). For the following values of parameters [4, 6]: $\mu_\nu = 2.9 \times 10^{-11}\mu_B, \Delta m^2 = 7.6 \times 10^{-5} eV^2, n^{eff} = 10^{24} cm^{-3}, E = 1GeV$ we get $\xi = 1.1 \times 10^5/B(G)$. In the region of small fields ($\sim$ few hundred gauss) $\xi \gg 1$ and we get $\gamma_g \approx 2\pi$. Thus the effect of geometric phase will not be significant in this region. On the other hand in the region of extremely large fields ($> 10^7$ gauss) $\xi \ll 1$ and $\gamma_g \approx \pi$. Thus if the magnetic field is sufficiently strong the neutrino state can develop large geometric phase which can influence neutrino spin precession and lead to resonant spin conversion $\nu_L \to \nu_R$ as noticed in [5].
Also as can be seen from (10) the presence of matter term results in reduction of γ_g, thus large values of matter potential V leads to suppression of spin conversion.

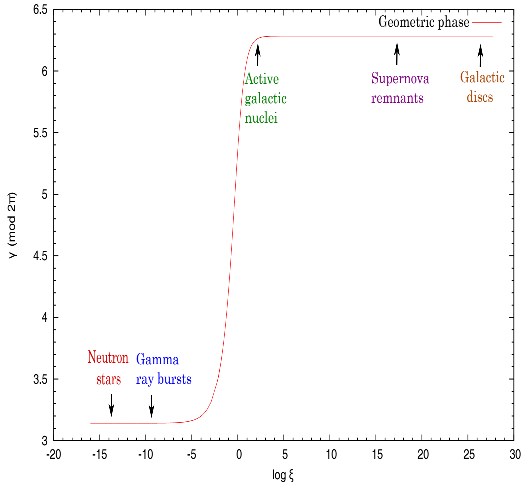

FIG. 1: Geometric phase variation with $\log\xi$. The dimensionless quantity ξ can be interpreted as the ratio of matter potential of neutrino interaction to the energy difference between its eigenstates were it to propagate in vacuum. Propagation in various astrophysical objects with vastly different magnetic fields ($\sim 10^{12}G$ for Neutron stars to $\sim 10^{-7}G$ for galactic discs) results in geometric phase of neutrino eigenstate to vary from π to 2π.

References

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