Crustal fraction of moment of inertia in pulsars

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Introduction

Pulsar glitches, which are discontinuities in the spin-down of pulsars, involve sudden transfer of angular momentum from an isolated component (consisting of superfluid neutrons in crust) to the entire star through vortex unpinning. The sudden jumps in rotational frequencies ω which may be as large as $\frac{\Delta\omega}{\omega}\sim10^{-6}$-$10^{-9}$ are observed for many pulsars [1]. The frequency of observed glitches is statistically consistent with the hypothesis that all radio pulsars experience glitches. Glitches are thought to originate from interactions between the rigid neutron star crust, typically somewhat more than a kilometer thick, and rotational vortices in a neutron superfluid.

In the present work, stability of the β-equilibrated dense nuclear matter is analyzed with respect to the thermodynamic stability conditions. Based on the density dependent M3Y (DDM3Y) [2] effective nucleon-nucleon (NN) interaction, the location of the inner edge of neutron star crusts and core-crust transition density and pressure are calculated and crustal fraction of moment of inertia is determined. These results for pressure and density at core-crust transition together with the observed minimum crustal fraction of the total moment of inertia provide a new limit for the radius of the Vela pulsar.

Core-crust transition and crustal fraction of moment of inertia

The quantity $V_{thermal}$ which determines the thermodynamic instability region of neutron star matter at β-equilibrium is given by $V_{thermal} = -(\frac{\partial P}{\partial v})_\mu$ which in terms of ρ, ϵ and x_p can be written as:

$$V_{thermal} = \rho^2\Big[2\rho\frac{\partial\epsilon}{\partial\rho} + \rho^2\frac{\partial^2\epsilon}{\partial\rho^2} - \frac{(\rho\frac{\partial^2\epsilon}{\partial\rho\partial x_p})^2}{\frac{\partial^2\epsilon}{\partial x_p^2}}\Big] \quad (1)$$

where ρ, ϵ and x_p are the baryonic number density, energy/baryon and β-equilibrium proton fraction, respectively [2]. The intrinsic stability condition of a single phase for locally neutral matter under β-equilibrium is determined, thermodynamically, by the positivity of the $V_{thermal}$, under constant chemical potential. However, the limiting density that breaks these conditions will correspond to the core-crust (liquid-solid) phase transition. Thus transition density ρ_t (with corresponding pressure P_t and proton fraction $x_{p(t)}$) is determined at which $V_{thermal} = 0$ and goes to negative with decreasing density.

The crustal fraction of the moment of inertia $\frac{\Delta I}{I}$ can be expressed as a function of M (gravitational mass of the star) and R (radius of the star) with the only dependence on the equation of state arising from the values of transition density ρ_t and pressure P_t. Actually, the major dependence is on the value of P_t, since ρ_t enters only as a correction in the following approximate formula

$$\frac{\Delta I}{I} \approx \frac{28\pi P_t R^3}{3Mc^2}\left(\frac{1-1.67\xi-0.6\xi^2}{\xi}\right) \times\left(1+\frac{2P_t}{\rho_t m_b c^2}\frac{(1+7\xi)(1-2\xi)}{\xi^2}\right)^{-1} \quad (2)$$

where $\xi = \frac{GM}{Rc^2}$. The angular momentum requirements of glitches in the Vela pulsar indicate that more than 0.014 of the moment of inertia drives these events. So, if glitches originate in the liquid of the inner crust, this means that $\frac{\Delta I}{I} > 1.4\%$.

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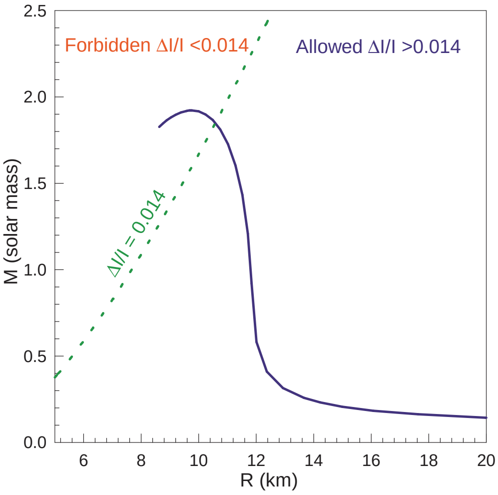

FIG. 1: Mass-radius relation of slowly rotating neutron stars for the present EoS. For Vela pulsar, constraint $\frac{\Delta I}{I}$>1.4% ⇒allowed masses and radii lie to the right of the line defined by $\frac{\Delta I}{I}$=0.014 (for ρ_t = 0.0938 fm^{-3}, P$_t$=0.5006 MeV fm^{-3}).

Calculations and Results

Calculations for masses and radii are performed using EoS covering the crustal region of a compact star which are FMT+BPS+BBP up to number density of 0.0582 fm^{-3} and β-equilibrated neutron star matter beyond. Once masses and radii are determined by solving TOV equation, $\frac{\Delta I}{I}$ are obtained from Eq.(2) and listed in Table-I. In Fig.-1, the mass-radius relation of slowly rotating neutron stars is shown. Using Eq.(2) again the mass-radius relation is obtained for fixed values of $\frac{\Delta I}{I}$, ρ_t and P$_t$. This is then plotted in the same figure for $\frac{\Delta I}{I}$ equal to 0.014. For Vela pulsar, the constraint $\frac{\Delta I}{I} > 1.4\%$ implies that allowed mass-radius lie to the right of the line defined by $\frac{\Delta I}{I} = 0.014$ (for $\rho_t = 0.0938$ fm^{-3}, P$_t$ = 0.5006 MeV fm^{-3}). This condition is given by inequality $R \geq 4.10 + 3.36 M/M_\odot$ kms.

Summary and Conclusion

The DDM3Y effective interaction which provides a unified description of elastic and inelastic scattering, proton, α, cluster radioactivities and nuclear matter properties, also provides an excellent description of the β-equilibrated neutron star matter which is stiff enough at high densities to reconcile with the recent observations of the massive compact stars while the corresponding symmetry energy is supersoft as preferred by the FOPI/GSI experimental data. The neutron star core-crust transition density and pressure along with observed minimum crustal fraction of the total moment of inertia of the Vela pulsar provide a new limit for its radius. It is somewhat different from the other estimates [3, 4] and imposes a new constraint $R \geq 4.10 + 3.36 M/M_\odot$ kms for the mass-radius relation of Vela like neutron stars.

TABLE I: Masses, radii & crustal fraction of moment of inertia as a function of central density ρ_c.

ρ_c	R	M	$\frac{\Delta I}{I}$
fm^{-3}	km	$M_\odot$	fraction
2.0000	8.6326	1.8273	0.0055
1.9000	8.7574	1.8463	0.0057
1.8000	8.8932	1.8646	0.0060
1.7000	9.0412	1.8820	0.0063
1.6000	9.2024	1.8975	0.0067
1.5000	9.3768	1.9103	0.0072
1.4000	9.5671	1.9190	0.0079
1.3000	9.7728	1.9219	0.0087
1.2000	9.9949	1.9165	0.0098
1.1000	10.2326	1.8994	0.0112
1.0000	10.4852	1.8662	0.0131
0.9000	10.7488	1.8111	0.0158
0.8000	11.0178	1.7265	0.0197
0.7000	11.2820	1.6035	0.0255
0.6000	11.5219	1.4333	0.0346
0.5000	11.7115	1.2083	0.0493
0.4000	11.8369	0.9204	0.0752
0.3000	12.0086	0.5805	0.1248
0.2000	13.6644	0.2578	0.2542
0.1500	19.9704	0.1439	0.6308

References

[1] A.G. Lyne *Pulsars:Problems and Progress*, S. Johnston et al., eds., 73 (ASP,1996).

[2] Debasis Atta and D. N. Basu, Phys. Rev. **C 90**, 035802 (2014).

[3] B. Link, R. I. Epstein and J. M. Lattimer, Phys. Rev. Lett. **83**, 3362 (1999).

[4] Jun Xu, Lie-Wen Chen, Bao-An Li, Hong-Ru Ma, Phys. Rev. **C 79**, 035802 (2009).