Evaluating Transport Coefficients of Hot Hadronic Matter

Sourav Sarkar*
Theoretical High Energy Physics Division, Variable Energy Cyclotron Centre,
1/AF Bidhannagar, Kolkata - 700064, INDIA

The space-time evolution of strongly interacting hot/dense matter is rather well described by relativistic viscous fluid dynamics; the effective theory for long wavelength, low frequency modes propagating in the medium. The dynamics of these modes are characterized by transport coefficients which describe the response of the system to fluctuations which occur in systems perturbed slightly out of equilibrium. The hydrodynamic equations imply conservation of the locally conserved currents i.e. baryon number and stress energy tensor. Hence, fluctuations in number, energy and momentum densities are the hydrodynamic modes and thermal conductivity (κ), shear viscosity (η) and bulk viscosity (ζ) characterize the hydrodynamic response. Though the hydrodynamic equations may be derived from entropy considerations using the second law of thermodynamics, a microscopic approach is necessary to determine the transport coefficients.

There are two basic methods to calculate the transport coefficients: response theory and transport theory. In the response approach one uses standard perturbation theory to obtain the transport coefficients in terms of retarded Green functions which are then evaluated using equilibrium quantum field theory. Though this approach is more fundamental, its implementation is difficult since one has to resum infinite sets of diagrams to even obtain the leading order behavior. On the other hand, the transport approach appears more tractable. Here one expresses the non-equilibrium phase space distribution function as an expansion around the local equilibrium distribution, the deviation function being expressed as a linear combination of the gradients with unknown coefficients. The transport coefficients can then be expressed in terms of these momentum dependent coefficient functions. In order to obtain these functions one has to solve the linear integral equations which result on using the non-equilibrium distribution function on both sides of the Boltzmann equation. This is the Chapman-Enskog method. A standard procedure of obtaining the unknown coefficient functions is to expand them in terms of orthogonal polynomials and convert the integral equation into algebraic ones.

In addition to the coefficients of viscosity and thermal conductivity the corresponding relaxation times (τ) also go as input in the viscous hydrodynamic equations. They indicate the time taken by the fluxes to relax to their steady state values and consequently play an important role in determining the space-time evolution of relativistic heavy ion collisions. This is more so for systems where τ is of the same order or larger than the mean collision time of the particles since several collisions may occur during the relaxation of the dissipative flows to their steady state values as in the case of a strongly interacting system like the one created in heavy ion collisions. As is well known, the Chapman-Enskog approach leads to a linear relationship between the thermodynamic forces and the corresponding irreversible flows. Because of the parabolic nature of the equations of motion this results in infinite speeds of these flows. In order to have access to the relaxation times we use the more general 14-moment method due to Grad. With the inclusion of the viscous pressure tensor and the heat flow to the original (five) hydrodynamic variables the relations between fluxes and forces contain time derivatives of the fluxes and cross-couplings between

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*Electronic address: `sourav@vecc.gov.in`

them. The hyperbolic nature of the equations of motion in this case result in finite relaxation times of the dissipative flows.

In the transport theory approach the (differential) scattering cross-section in the collision integral is the dynamical input and plays a significant role in determining the magnitude of the transport coefficients. For bosons, medium effects affect the collision integral in two competing ways. The larger phase space occupancy due to the Bose factors $(1+f)(1+f)$ for the final state particles results in an increase of the collision rate. This is compensated to some extent by a smaller effective cross-section on account of many-body effects. For fermions in the final state the enhancement factors are replaced by Pauli blocking.

In view of the above we investigate the effect of the medium on the temperature dependence of the transport coefficients considering the case of a hot pion gas. To evaluate the scattering amplitude we adopt an effective interaction involving ρ and σ meson exchange. The meson propagators in the s-channel processes are replaced with effective ones obtained by a Dyson-Schwinger sum of one-loop self-energy diagrams involving the pion. The isospin averaged amplitude is found to agree very well with the estimate based on measured phase-shifts. The self energy diagrams are then evaluated at finite temperature to obtain the in-medium cross-section. For the σ meson only the $\pi\pi$ loop diagram is calculated in the medium whereas in case of the ρ meson in addition to the $\pi\pi$ loop graph, $\pi\omega$, πh_1, πa_1 self-energy graphs are evaluated using interactions from chiral perturbation theory. In relativistic heavy ion collisions, below the crossover temperature inelastic reactions cease and this leads to chemical freeze-out of hadrons. Since only elastic collisions occur the number-density gets fixed at this temperature and to conserve it a phenomenological chemical potential is introduced which increases with decreasing temperature until kinetic freeze-out is reached. With these inputs we obtain an increase in the imaginary part of the self-energy due to scattering and decay processes in the medium which results in enlarged widths of the exchanged ρ and σ. This is manifested in a suppression of the magnitude of the cross-section and a small downward shift in the peak position as a function of the c.m. energy.

Using the in-medium differential scattering cross-section we obtain the shear and bulk viscosities and thermal conductivity for a pion gas. We do observe a significant effect of the medium when these coefficients are plotted as a function of the temperature of the medium. Similar effects were also seen for the case of the relaxation times of the dissipative flows.

Next we consider a gas of pions and nucleons. In this case we have to deal with a coupled set of equations; one for the pion and another for the nucleon. The collision integral for the pions involve both $\pi\pi$ and πN scattering. The latter is known to be dominated by the Δ baryon. The in-medium πN scattering amplitude obtained by using the Δ self-energy evaluated from πN, ρN and $\Delta\pi$ loops shows a significant suppression compared to vacuum.

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