Quark Number Susceptibility : Revisited in mean field theories

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Introduction

In relativistic heavy-ion collisions the fluctuations and correlations of conserved charges are considered to carry promising signals for the formation of the exotic quark gluon plasma (QGP). The characteristics of quark-hadron phase transition can be understood by analyzing the fluctuations of the system. Fluctuations of conserved quantum numbers are associated with the corresponding susceptibilities because of the symmetry of the system. The underlying fact is that these fluctuations as defined through the static correlators become identical to the direct calculation of these susceptibilities defined through the thermodynamic derivatives, due to the fluctuation-dissipation theorem. In effective approaches like NJL or PNJL models the Quark Number Susceptibility (QNS) is usually obtained as the second order Taylor coefficient of pressure when it is Taylor expanded in the direction of the quark chemical potential, μ_q i.e. $\chi_q = \frac{\partial^2 P}{\partial \mu_q^2}$. In model calculations any response of a thermodynamic quantity to some external parameters should also account for the fact that the mean fields also depend implicitly on those external parameters. Therefore, proper care has to be taken to relate the thermodynamic derivatives (viz., QNS) with the fluctuation associated with the conserved density. In the present work through an extensive exercise we have shown the fluctuation-dissipation theorem holds true within the framework of NJL and PNJL models.

QNS from FDT

According to the fluctuation-dissipation theorem (FDT), the QNS can be obtained from the time-time component of the current-current correlator in the vector channel. The QNS is then expressed as

$$\begin{aligned}\chi_q &= \frac{\partial\rho}{\partial\mu} = \frac{\partial^2\mathcal{P}}{\partial\mu^2} = \int d^4x \langle J_0(0,\vec{x}) J_0(0,\vec{0})\rangle \\ &= -\lim_{l\to 0} \mathrm{Re}\Pi_{00}(0,l) \\ &= \lim_{l\to 0} \beta \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{-2}{1-e^{-\beta\omega}} \mathrm{Im}\Pi_{00}(\omega,l),\end{aligned}$$

where J_0 is the temporal component of the vector current and $\Pi_{00}(\omega,l)$ is the time-time component of the vector correlation function or self-energy with external four momenta $L \equiv (\omega, l = |\vec{l}|)$. In the mean field model, the thermodynamic potential is a functional of the mean field $\sigma(m_0,T,\mu_q)$, and is given as [1],

$$\Omega[\sigma, m_0, T, \mu_q] = -i\mathrm{Tr}[\ln {S_1}^{-1}] + \frac{G}{2}\sigma^2. \quad (1)$$

S_1 being the dressed propagator for modified quark mass $M = m_0 - G\sigma$ [1], The second term in Ω may be considered as the background contribution of the mean field σ. In the imaginary time formalism, at finite temperature and chemical potential, the fourth component of momentum becomes $p_0 = i(2n+1)\pi T + \mu_q$ and thus applying Ward identity,

$$\frac{d{S_1}^{-1}}{d\mu_q} = \gamma_0 + \left(G\frac{d\sigma}{d\mu_q}\right)\cdot \mathbb{1}_D \equiv \Gamma_0, \quad (2)$$

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Γ_0 being the effective three point function. Starting from the definition of σ,

$$\sigma = i\mathrm{Tr}(S_1).$$

we have,

$$\frac{d\sigma}{d\mu_q} = -i\mathrm{Tr}[S_1\Gamma_0 S_1]$$
$$= -i\mathrm{Tr}(S_1\gamma_0 S_1) - i\mathrm{Tr}\Big(S_1 G\frac{d\sigma}{d\mu_q}S_1\Big),$$

The detail calculation can be found in [1]. Since in the NJL Lagrangian the explicit interaction term through chiral condensate σ is present, this would contribute to the physical quantities one would like to compute. Let us avoid this computation and start from the thermodynamic derivative of pressure. Since mean fields are also dependent on external parameters, in model calculation we have to take the total derivative rather than the explicit one. It can be shown that we will arrive at [1]

$$\chi_q = -i\mathrm{Tr}(\Gamma_0 S_1\Gamma_0 S_1) - G(-i\mathrm{Tr}[S_1\Gamma_0 S_1])^2. \quad (3)$$

The right hand side of (3) can be viewed in terms of diagrammatic topology as displayed in Fig.1. It is evident that these are equivalent to the vector correlator in NJL model in static limit or amputed legs as given in Fig.1. This implies that the inclusion of implicit μ_q dependences of the mean fields is not ad hoc, rather it enables us, from the field theoretic point of view, to compute the correlators associated with the conserved density fluctuation through diagrammatic way in NJL model.

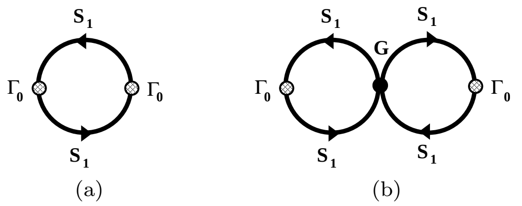

FIG. 1: Static or amputed vector correlators with modified propagator and effective three-point function.

In the present form of the PNJL model, it is difficult to draw the exact form of vector correlator as we have no gluon-like quasi particles in PNJL model.

References

[1] S. K. Ghosh, A. Lahiri, S. Majumder, M. G. Mustafa, S. Raha, and R. Ray, Phys. Rev. **D90**, 054030 (2014).