Sensitivity analysis of optimized nuclear energy density functional

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Being the exact nature of nuclear force unknown, parameters of nuclear models have been optimized by fitting different kind of data e.g. properties of finite nuclei as well as neutron stars. As the information about exact correspondence between parameters of the model and the fitted data are missing, it has led to a plethora of nuclear models. These informations can be extracted by studying correlations between different parameters and fitted data in all sorts of combinations within the framework of covariance analysis. This not only minimizes the number of the parameters of the model, but also helps to restrict unnecessary addition of redundant data. First, a model is obtained by optimizing the objective function $\chi^2(\mathbf{p})$ as given by,

$$\chi^2(\mathbf{p}) = \frac{1}{N_d - N_p} \sum_{i=1}^{N_d} \left(\frac{\mathcal{O}_i^{exp} - \mathcal{O}_i^{th}(\mathbf{p})}{\Delta \mathcal{O}_i} \right)^2, \tag{1}$$

where, N_d and N_p are the number of experimental data points and the number of fitted parameters, respectively. $\Delta\mathcal{O}_i$ is the adopted error and $\mathcal{O}_i^{exp}$ and $\mathcal{O}_i^{th}(\mathbf{p})$ are the experimental and the corresponding theoretical values for a given observable. Once the optimized parameter set is obtained the correlation coefficient between two quantities $\mathcal{A}$ and $\mathcal{B}$, which may be a parameter as well as an observable, can be evaluated within the covariance analysis as,

$$c_{\mathcal{AB}} = \frac{\overline{\Delta\mathcal{A}\,\Delta\mathcal{B}}}{\sqrt{\overline{\Delta\mathcal{A}^2}\;\overline{\Delta\mathcal{B}^2}}}, \tag{2}$$

where $\overline{\Delta\mathcal{A}\,\Delta\mathcal{B}}$ is the covariance between $\mathcal{A}$ and $\mathcal{B}$ and $\overline{\Delta\mathcal{A}^2}$ and $\overline{\Delta\mathcal{B}^2}$ are mean square errors on them, respectively, which can be obtained by the help of inverted curvature matrix, elements of which are given by,

$$\mathcal{C}_{\alpha\beta} = \frac{1}{2}\Big(\frac{\partial^2\chi^2(\mathbf{p})}{\partial \mathrm{p}_\alpha \partial \mathrm{p}_\beta}\Big)_{\mathbf{p_0}}. \tag{3}$$

After doing all these, still one very important information remains unknown, which is, how much sensitive one particular datum is to a particular parameter. Only by knowing that information, one can justify a new type fit data added in the process of optimizing the parameters of a nuclear energy density functional. The sensitivity of a given parameter to a particular data can be determined in terms of the sensitivity matrix of dimension $N_p \times N_d$ defined as [1],

$$S(\mathbf{p}) = [\hat{J}(\mathbf{p})\hat{J}^T(\mathbf{p})]^{-1}\hat{J}(\mathbf{p}). \tag{4}$$

Here $\hat{J}(\mathbf{p})$ is the Jacobian matrix with the same dimension as $S(\mathbf{p})$; its elements are given as,

$$\hat{J}_{\alpha i} = \frac{1}{\Delta\mathcal{O}_i}\left(\frac{\partial\mathcal{O}_i}{\partial p_\alpha}\right)_{\mathbf{p_0}}. \tag{5}$$

The sensitivity of the parameter p_α to the i-th data is given by $S^2_{\alpha i}$ which is normalized to $\sum_{i=1}^{N_d} S^2_{\alpha i} = 1$. The relative sensitivity for a subset of data can likewise be obtained by summing $S^2_{\alpha i}$ over that subset.

In this contribution we will show that if one has an optimized set of model parameters, sensitivity analysis can be performed that to reveal the quantitative idea about how different quantities of interest are sensitive to some specific data chosen to optimize the energy density functional. In particular, we want to study sensitivity of binding energy of extremely asymmetric nuclei to different symmetry energy parameters. To perform this we have constructed two relativistic mean field (RMF) models namely, SINPB and SINPA [2]. In model SINPB we have chosen binding energies and charge radii of some standard set of nuclei (^{16}O, ^{40}Ca, ^{48}Ca, ^{56}Ni, ^{68}Ni, ^{90}Zr, ^{100}Sn, ^{116}Sn, ^{132}Sn, ^{144}Sm and ^{208}Pb) along with some moderately asymmetric nuclei e.g. ^{54}Ca, ^{78}Ni and ^{138}Sn (neutron to proton ratio ~ 1.7). In SINPA binding energy of some extremely asymmetric nuclei (neutron to proton ratio ~ 2) ^{24}O, ^{30}Ne, ^{36}Mg and ^{58}Ca along with maximum mass of neutron star (M_{max}) are added to the data set in the base model SINPB.

Different nuclear matter properties along with the errors on them calculated for SINPB and SINPA are given in Tab. I. A very good

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TABLE I: Different nuclear matter properties: the binding energy per nucleon for symmetric matter $\mathcal{E}_0$, incompressibility coefficient K_0, Dirac effective mass of nucleon M_0^* (scaled by nucleon mass M), C_{sym}^0 and L_0 evaluated at saturation density ρ_0 along with the errors on them for the models SINPB and SINPA. One can observe the improvements on the errors in SINPA where the binding energies of extremely asymmetric nuclei along with M_{max} are additionally taken as fit data.

Observable	SINPB	SINPA
$\mathcal{E}_0$ (MeV)	-16.04 ± 0.06	-16.00 ± 0.05
K_0 (MeV)	206 ± 20	203 ± 6
ρ_0 (fm^{-3})	0.150 ± 0.002	0.151 ± 0.001
M_0^*/M	0.59 ± 0.01	0.58 ± 0.01
C_{sym}^0 (MeV)	33.95 ± 2.41	31.20 ± 1.11
L_0 (MeV)	71.55 ± 18.89	53.86 ± 4.66

overall improvement is seen for the errors on different quantities for the model SINPA compared to SINPB. It is very similar to the results observed in Ref. [3].

To know how the additional data in SINPA helped to constrain different nuclear matter properties, especially the symmetry energy parameters, we performed the sensitivity analysis on SINPA, first in the usual way, by making three subset of data, binding energies, charge radii and M_{max} to different nuclear matter properties in Fig. 1. However, apart

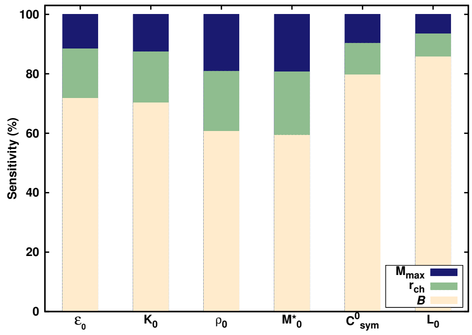

FIG. 1: Relative sensitivity of different nuclear matter parameters to three groups of fit data, namely, binding energies (B), charge radii (r_{ch}) and M_{max}.

from very high sensitivity of M_{max} to ρ_0 and M_0^* no other specific information can be extracted from this figure.

To reveal the information content of those highly asymmetric nuclei we plotted the relative sensitivity of the same set of data to the nuclear matter properties but by making the the three groups is a different way in Fig 2. In the first group we have taken binding energy of only those four highly asymmetric nuclei (^{24}O, ^{30}Ne, ^{36}Mg and ^{58}Ca). Second group consists of rest of the data on finite nuclei. Third group consists of lone data on neutron star, M_{max}. Though, the group consisting of extremely asymmetric nuclei have only four data, they control almost $\sim 40\%$ of the symmetry energy elements like C_{sym}^0 or L_0. This clearly figures out how those extremely asymmetric nuclei along with M_{max} helped to constrain the different properties of nuclear matter, especially the symmetry energy elements in SINPA compared to SINPB.

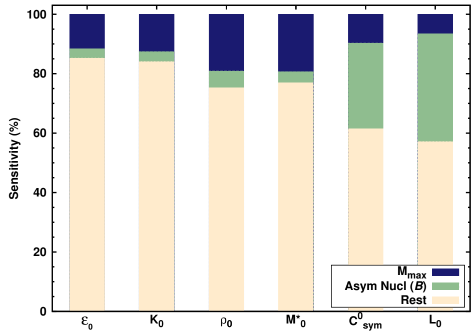

FIG. 2: Same as Fig. 1, but, with different grouping of the fit data of finite nuclei. See text for details.

To sum up, we have made an investigation in this contribution on the extraction of the precision information of exact correspondence between experimental data and properties of nuclear matter. To perform that we applied the method of sensitivity analysis. A comparative study of the covariance analysis of different nuclear matter properties made with two RMF models SINPB and SINPA (which included further data from extremely asymmetric nuclei with neutron to proton ratio ~ 2 and the observed maximum mass M_{max} of neutron star) shows that the nuclear symmetry energy properties are determined in much narrower constraints from the latter model. The conclusion is further reinforced from the sensitivity analysis of the different nuclear matter parameters to the experimental data set taken for such an analysis.

References

[1] M. Kortelainen *et al* , Phys. Rev. C. **82**, 024313 (2010).

[2] C. Mondal *et al* , Phys. Rev. C. **93**, 044328 (2016).

[3] C. Mondal *et al* , Phys. Rev. C. **92**, 024302 (2015).