Signature partner pairs of superdeformed rotational bands in $^{192}$Tl

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Introduction

Rotational bands associated with superdeformed (SD) shapes in $A \sim 190$ mass region are identified by the dynamic moment of inertia ($\mathfrak{I}^{(2)}$), which rises smoothly with the rotational frequency ($\hbar \omega$). This property of SD bands are only observed in the $A \sim 190$ mass region. In lower mass region of $A \sim 130, 150$, pronounced variation of $\mathfrak{I}^{(2)}$ with $\hbar \omega$ is seen. This distinguishing feature of $\mathfrak{I}^{(2)}$ is the result of gradual alignment of $i_{13/2}$ protons and $j_{15/2}$ neutrons. Another characteristic feature of the $A \sim 190$ mass region is that these bands are observed at lower ($\sim 10 \hbar$) spins and has smaller transition energies when compared with $A \sim 80, 150$ mass region ($> 20 \hbar$). This provide the unique opportunity to study the second potential well. Prediction of $^{192}$Hg as the “doubly magic” SD nucleus based on the large gaps in Woods-Saxon single-particle diagram at large deformation supported the fact that multiple SD bands have been found in the neighbouring Hg isotopes [1–4]. Further it was noticed that the various SD bands observed are identical to this “doubly magic” $^{192}$Hg SD nucleus. These similarities in the transition energies are explained in terms of pseudo-spin symmetry of SD nuclei [5]. Out of many interesting properties of SD bands observed in $A \sim 190$ mass region, another astonishing property is the observation [6, 7] of “flat bands” in $^{192}$Tl, where $\mathfrak{I}^{(2)}$ is observed to be constant with the $\hbar \omega$ in the two bands.

Presently, many theoretical models like Harris $\omega^2$ expansion [8], $ab$ expression [9], variable moment of inertia model [10] etc. are available which provide the reliable spins of SD bands. In the present approach, we have calculated band head MoI of SD bands available in $^{192}$Tl in $A \sim 190$ mass region using soft rotor formula (SRF).

Formalism

A nuclear softness (NS) formula was proposed by Gupta [11]. Later Brentano et al. [12] given the similar expression for well-deformed nuclei and nuclei in transitional region. Brentano et al. called this as “soft-rotor formula” (SRF).

The energy formula for a rigid rotator is given by

$$ E = \frac{\hbar^2}{2I} (I + 1). \quad (1) $$

This formula predicts state largely higher than obtained from experiments. The variation of MoI with the angular momentum was incorporated and modified Eq. (1) as,

$$ E = \frac{\hbar^2}{23I} (I + 1). \quad (2) $$

After Taylor series expansion of $\mathfrak{I}_I$ about its ground state value $\mathfrak{I}_0$ for $I = 0$ and representing in terms of “Softness” parameter ($\sigma$), we get,

$$ E_I = \frac{\hbar^2 I(I + 1)}{23\mathfrak{I}_0} \times \left( 1 - \frac{\sigma_1 I^2}{1 + \sigma_1 I + \sigma_2 I^2} \right) \times \left( 1 - \frac{\sigma_2 I^2}{1 + \sigma_1 I + \sigma_2 I^2} \right) \cdots \quad (3) $$

where, $\sigma_1, \sigma_2, \sigma_3, \ldots$ are the constants of first, second, third etc., orders of “nuclear softness”.

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Keeping the nuclear softness to only first order i.e putting $\sigma_2, \sigma_3, \ldots = 0$, we get a two parameter formula. Eq. (3) can be written as $(\sigma = \sigma)$,

$$E = \frac{\hbar^2}{2\Im_0} \times \frac{I(I+1)}{1+\sigma I}.$$  \hspace{1cm} (4)

where, $\Im_0$ and $\sigma$ are the fitting parameters. Since, intraband energies and intensities are the only spectroscopic properties whose information are available for superdeformed bands hence one may choose to fit $E_\gamma$ transitions as.

$$E_\gamma = E(I) - E(I-2).$$ \hspace{1cm} (5)

Using Eq. (4) and Eq. (5) the transition energies for superdeformed bands is expressed as

$$E_\gamma(I) = \frac{\hbar^2}{2\Im_0} \times \left[ \frac{I(I+1)}{1+\sigma I} \right] - \frac{(I-2)(I-1)}{1+\sigma(I-2)}.$$ \hspace{1cm} (6)

The parameters $\Im_0$ and $\sigma$ are obtained by least-squares fitting.

<table>
<thead>
<tr>
<th>SD band</th>
<th>$E_\gamma(I_0 + 2 \to I_0)$ (keV)</th>
<th>$\Im_0$ ($\hbar^2$ MeV$^{-1}$)</th>
<th>$\sigma$ ($\times 10^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{192}$Tl(1)</td>
<td>283.0</td>
<td>102.73</td>
<td>0.4514</td>
</tr>
<tr>
<td>$^{192}$Tl(2)</td>
<td>337.5</td>
<td>102.88</td>
<td>3.3932</td>
</tr>
<tr>
<td>$^{192}$Tl(3)</td>
<td>233.4</td>
<td>94.45</td>
<td>20.521</td>
</tr>
<tr>
<td>$^{192}$Tl(4)</td>
<td>213.4</td>
<td>94.45</td>
<td>20.598</td>
</tr>
</tbody>
</table>

### Results and Discussion

Observed transition energies of $^{192}$Tl[1, 2, 3, 4], indexed in the table of SD bands [13] and continuously updated ENSDF database [14] have been fitted to SRF model. The values of parameters $\Im_0$ and $\sigma$ is obtained by fitting of $E_\gamma$ transition energies in Eq. 6. The calculated band head MoI with SRF formula are almost identical for $^{192}$Tl(1), $^{192}$Tl(2) and $^{192}$Tl(3), $^{192}$Tl(4) (See Table I).

### Conclusion

At low transition energies, the intraband $\gamma$-transitions of one band is close to midpoint energies of adjacent transition of other band suggest that these bands are two pair of signature partner. Identical $(\delta \Im_0/\Im_0 \approx 10^{-3})$ band head MoI obtained using SRF formula for $^{192}$Tl(1), $^{192}$Tl(2) and $^{192}$Tl(3), $^{192}$Tl(4) verified the experimentally observed signature partners.

### Acknowledgments

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### References