A simple yet efficient correction to the Bethe-Weizsäcker semi-empirical mass formula

Bijan Kumar Gangopadhyay[1] *
[1]Department of Physics, Sovarani Memorial College, Jagatballavpur, Howrah - 711408, INDIA
* email: bkgangopadhyay@gmail.com

Introduction

The semi-empirical mass formula introduced by Bethe and Weizsäcker (BW) gives a reasonable estimate of the nuclear binding energies for the medium and the heavy nuclei but this formula underestimates the values of the nuclear binding energies of the light nuclei to a great extent. So, a correction to the BW formula was much needed to predict the experimental values of the binding energies more precisely through a theoretical prescription. Several attempts were made in the last decade to improve the theoretical predictions of the binding energies with respect to the experimental findings. Samanta and Adhikari proposed a modification in the symmetric and the pairing energy terms in the BW formula through a phenomenological study [1]. Later Royer made a correction to the volume and the surface energy terms present in the BW formula [2]. The author also introduced several additional correction terms in the BW formula such as the shell energy, the curvature energy, the Wigner energy and the congruence energy which are expected to be present in a nucleus according to the liquid drop model, along with a proton form factor correction due to the finite size of the protons. Although this theoretical model indeed improves the predictability of the binding energy, this formula assumes a very intricate form along with its own limitations and leaves a scope for further modifications, if possible, with a more simplistic approach. In the present work we have proposed a significantly simple theoretical model which modifies the BW formula by empirically combining all the major corrections introduced by Royer into a single correction term. We will show that our simplistic correction to the BW formulation predicts the binding energies for a wide range of nuclei having atomic mass number (A) between 16 and 209 significantly close to the experimental values with notable improvements in the results for the lighter and the heavier nuclei.

Theory

The BW mass formula for the binding energy (E_B) of a nucleus with proton number Z and nuclear mass number A can be written as,

$$E_B = C_1 A - C_2 A^{\frac{2}{3}} - C_3 \frac{Z(Z-1)}{A^{\frac{1}{3}}} - C_{sym} \frac{(A-2Z)^2}{A} + \delta$$

where the terms (from the left to the right) represent the volume energy, the surface energy, the Coulomb energy, the asymmetric energy, and the pairing energy respectively. The pairing energy is given by $11/A^{1/2}$ MeV and it is positive for the even-even nuclei, negative for the odd-odds and zero for the nuclei with odd A. The constants were taken as C_1=15.79 MeV, C_2=18.34 MeV, C_3=0.71 MeV and C_{sym}=23.21 MeV. This empirical relation was in good agreement with the experimental binding energies for the heavy nuclei but differs widely for the light nuclei. Royer introduced several additional energy terms in the BW formula such as the shell energy, the curvature energy, Weigner energy and the congruent energy [2]. The author also made corrections to the volume and the surface term by expressing them in the powers of (N-Z)/A and also added an extra term, which is known as the form factor and proportional to Z^2/A, as a correction to the Coulomb energy considering the finite sizes of the protons. Taken together, these corrections yield a more rigorous and complex version of the BW formula. Here in this paper we propose an empirical correction to the BW formula which is significantly simple yet captures the major aspects of the corrections introduced by Royer through one single tuneable parameter and it is given as

$$E_B = C_1 A - C_2 A^{\frac{2}{3}} - C_3 \frac{Z(Z-1)}{A^{\frac{1}{3}}} - C_{sym} \frac{(A-2Z)^2}{A} + \delta +$$
$$+ C_f \frac{Z^2}{A} + C_{sh} A - K(A-2Z)(A^{-\frac{1}{3}} - A^{-1} - 1)$$

where $C_f Z^2/A$ denotes the correction due to the proton form factor and C_{sh} represents the shell energy per nucleon [4]. Here the nuclei are divided in to four groups according to their mass numbers and the value of K is tuned phenomenologically to compare best with the experimental results. The values of K are as follows: Group 1 [A=16-79], K=1.79; Group 2 [A=80-119], K=1.15; Group 3 [A=120-159], K=0.90; Group 4 [A=160-209], K=0.79.

Results and Discussions

Fig. 1 (upper panel) represents the variation of the binding energy per nucleon with the nuclear mass number. The present theory is in very good agreement with the experimental values for all the nuclei studied. Fig. 1 (lower panel) also shows an indicative plot to compare the results obtained from the present theory with the original BW

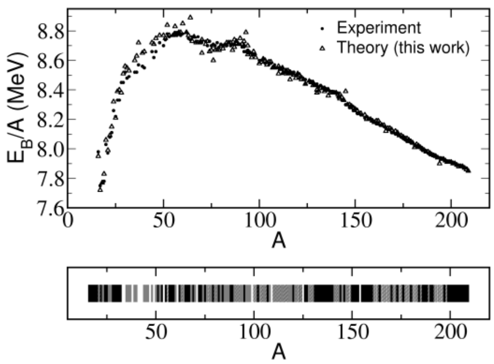

Fig. 1 Upper panel: Comparison of the binding energy per nucleon predicted from the present theory and the experimental values. Lower panel: Comparison between the present theory and the BW theory (see text for details).

formula. Here in this plot an improvement in the prediction of the E_B value with the present theory over the BW formula is shown with a black spectrum for the corresponding A value. Similarly, a white spectrum denotes where the original BW formula works better than the present theory. Equal results (within a precision up to 2 decimal points in E_B/A) from both the formulations are indicated by grey. From this plot it can be seen that the present theory predicts better results than the original BW theory for the majority of the nuclei. In particular, the present theory works significantly better for the heavy nuclei (A>100) and for the light nuclei (A<35) where BW theory shows larger deviations from the experimental values. Further we would like to point out few particular nuclei where the present theory shows remarkable improvement over the original BW theory. For oxygen (16), neon (20), neon (22) and magnesium (24) the present theory shows dispersion from the experiments within 0.06 MeV with an improvement in the results ranging from 0.15 to 0.25 MeV over the BW formula. However, for A between 50 and 75 the BW theory works more efficiently than the present theory.

The present theory predicts extra stability for some nuclei, which are not at their shell-closure of the nucleons, with respect to their nearby isotopes. For example, $_{28}Ni^{60}$ (N, number of neutrons=32), $_{26}Fe^{58}$ (N=32), $_{12}Mg^{26}$ (N=14), $_{13}Al^{27}$ (N=14) and $_{14}Si^{29}$ (N=15) are predicted to have higher binding energies than their nearby isotopes. Thus, according to the present theory, the nuclei with neutron or proton number 14 and 32 show extra stability. This may be due to the subshell closures of the nucleons as, according to the nuclear shell model, 14 and 32 can be expressed as "shell 2 + $2d_{5/2}$" and "shell 3A + $2p_{3/2}$". However, it takes further investigations to draw any specific conclusion about the presence of any new magic number such as 14 or 32 along with the established magic numbers 2, 8, 20, 28, 50, 82 and 126 which occur due to the proper shell closures of the nucleons.

Conclusions

The empirical formula presented in this paper predicts the binding energies for the nuclei with mass numbers ranging from 16 to 209 in very good agreement with the experimental values. In this regard, a significant improvement over the original BW formula has been found, especially for the heavy nuclei.

References

[1] C. Samanta and S. Adhikari, Phys. Rev. C 65, 037301 (2002).
[2] G.Royer, arXiv:0710.2503v1 [nucl-ex].
[3] G. Audi, A. H. Wapstra, Nucl. Phys. A, 565, 1 (1993).
[4] W. D. Myres, W. J. Swiatecki, LBL Report 36803,1994.