Search For Λ Shell Closures in Multi-Λ Hypernuclei

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Introduction

The quest for searching the magic numbers in strangeness nuclear physics is motivated from their discovery in conventional nuclear landscape.There are several signatures that characterize the magic numbers in hypernuclear physics.The magic numbers in nuclei are characterized by a large shell gap in single-particle energy levels which implies that the nucleon/hyperon in the lower level has comparatively large value of energy than that in higher level and hence imparts more stability to the nucleon/hyperon in the lower energy level.The extra stability imparted to a nuclear many body system by injection of hyperons can be estimated from the sudden fall in separation energy. The Λ- separation energy(one- and two-Λ) are considered to be the key quantities that reveals the nuclear response with the injection of strangeness degree of freedom.In order to make a clear prediction of magic numbers in hypernuclear regime two-lambda shell gap is a very useful quantity to calculate. In this paper, we made an attempt to look for the magic numbers in multi-lambda hypernuclei$^{16+n\Lambda}$O,$^{48+n\Lambda}$Ca,$^{58+n\Lambda}$Ni,$^{90+n\Lambda}$Zr, $^{124+n\Lambda}$Sn,$^{132+n\Lambda}$Sn,$^{208+n\Lambda}$Pb,$^{292+n\Lambda}$120, $^{304+n\Lambda}$120,$^{378+n\Lambda}$120 with doubly magic nucleonic core within the ambit of self-consistent relativistic mean-field framework with effective ΛN and ΛΛ interactions.

Formalism

Relativistic mean field theory has established itself as a promising framework to study the structural properties of normal as well as hypernuclei [1] and its extension to hypernuclei is accomplished by incorporating the the lambda-baryon interaction Lagrangian with effective ΛN potential.The total Lagrangian density for single lambda is reported in [2, 3]. In order to extend the study to multihypernuclei additional strange scalar σ^* and vector mesons ϕ have to be included which simulate the ΛΛ interaction [4]. Hence the total effective Lagrangian density can be written as

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\Lambda + \mathcal{L}_{\Lambda\Lambda}, \tag{1}$$

where $\mathcal{L}_N$, $\mathcal{L}_\Lambda$, $\mathcal{L}_{\Lambda\Lambda}$ are given by

$$\begin{aligned}\mathcal{L}_N &= \bar{\psi}_i\{i\gamma^\mu\partial_\mu - M\}\psi_i + \frac{1}{2}(\partial^\mu\sigma\partial_\mu\sigma - m_\sigma^2\sigma^2)\\ &- \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - g_s\bar{\psi}_i\psi_i\sigma - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu}\\ &+ \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - g_\omega\bar{\psi}_i\gamma^\mu\psi_i\omega_\mu - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}\\ &+ \frac{1}{2}m_\rho^2\vec{\rho}^\mu\vec{\rho}_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - g_\rho\bar{\psi}_i\gamma^\mu\vec{\tau}\psi_i\vec{\rho^\mu}\\ &- e\bar{\psi}_i\gamma^\mu\frac{(1-\tau_{3i})}{2}\psi_i A_\mu\end{aligned} \tag{2}$$

$$\begin{aligned}\mathcal{L}_\Lambda &= \bar{\psi}_\Lambda\{i\gamma^\mu\partial_\mu - m_\Lambda\}\psi_\Lambda - g_{\sigma\Lambda}\bar{\psi}_\Lambda\psi_\Lambda\sigma\\ &- g_{\omega\Lambda}\bar{\psi}_\Lambda\gamma^\mu\psi_\Lambda\omega_\mu\end{aligned} \tag{3}$$

$$\begin{aligned}\mathcal{L}_{\Lambda\Lambda} &= \frac{1}{2}(\partial^\mu\sigma^*\partial_\mu\sigma^* - m_{\sigma^*}^2\sigma^{*2}) - \frac{1}{4}S^{\mu\nu}S_{\mu\nu}\\ &+ \frac{1}{2}m_\phi^2\phi^\mu\phi_\mu - g_{\sigma^*\Lambda}\bar{\psi}_\Lambda\psi_\Lambda\sigma^* - g_{\phi\Lambda}\bar{\psi}_\Lambda\gamma^\mu\psi_\Lambda\phi_\mu\end{aligned} \tag{4}$$

where ψ and ψ_Λ denote the Dirac spinors for nucleon and Λ-hyperon, whose masses are M and m_Λ, respectively.The quantities m_σ, m_ω, m_ρ, m_{σ^*}, m_ϕ are the masses of σ, ω, ρ, σ^*,ϕ mesons and g_σ, g_ω, g_ρ, $g_{\sigma\Lambda}$, $g_{\omega\Lambda}$, $g_{\sigma^*\Lambda}$, $g_{\phi\Lambda}$. The non-linear self interaction coupling of σ mesons is denoted by g_2 and g_3. In the present

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calculations, we employed NL3* parametrization for meson-baryon coupling. In case of Λ-meson coupling, we adopt the nucleon coupling to hyperon coupling ratios defined as; $R_\sigma = \frac{g_{\sigma\Lambda}}{g_\sigma}$, $R_\omega = \frac{g_{\omega\Lambda}}{g_\omega}$, $R_{\sigma^*} = \frac{g_{\sigma^*\Lambda}}{g_\sigma}$ and $R_\phi = \frac{g_{\phi\Lambda}}{g_\omega}$. The values of the relative couplings used are $R_\omega = \frac{2}{3}$, $R_\phi = -\frac{\sqrt{2}}{3}$, $R_\sigma = 0.621$,and $R_{\sigma^*} = 0.69$ [6].

Results and conclusion

The size of the gap in the lambda spectrum is calculated by half of the difference in Fermi energy when going from a closed shell nucleus to a nucleus with two additional lambdas in an analogues way of determining neutron and proton shell gaps. This quantity is very well taken into account by two-lambda separation energy which is defined as the second difference of the binding energy.

$$\delta_{2\Lambda}(N,Z,A) = 2B.E.(N,Z,A) - B.E.(N,Z,\Lambda+2) - B.E.(N,Z,\Lambda-2) \quad (5)$$

$$\delta_{2\Lambda}(N,Z,A) = S_{2\Lambda}(N,Z,A) - S_{2\Lambda}(N,Z,\Lambda+2) \quad (6)$$

A peak of the two-lambda separation energy signifies the drastic change in it and thus serves as a potential signature of magic shell closure. Two lambda shell gap for all the multihypernuclei under study are shown in fig1.The lambda magic number predicted in various multihypernuclei under investigation are listed in table 1. A peak at certain Λ number indicates the existence of lambda shell closure.Whether the magic number is pronounced or not is dictated by the sharpness as well as magnitude of the peak. From the figure, it is clear that the magnitude of peak is found to be larger at$\Lambda = 2, 8, 14, 18, 20, 28, 34, 40, 50, 58, 70, 82$ indicating the lambda shell closure magic numbers. Moreover, a small peak also appeared at $\Lambda = 68$ due to closureness of subshell($2d_{\frac{3}{2}}$) making it a Λ semi-magic number. A peak with small magnitude appears at $\Lambda = 28$ representing a feeble magic number in contrast with the nucleon magic number. Further, the peaks are very pronounced at $\Lambda = 34$ and $\Lambda = 58$ indicating strong shell closures.The experimental confirmation of nucleonic shell closure of $N = 34$ [5] supports the prediction in hypernuclei case.Thus, it can be concluded that the Λ-magicity quite resembles with the nuclear magicity and it is expected that the predictions made in hypernuclear regime might serve as a significant input to make things clear regarding new nucleonic shell closures.

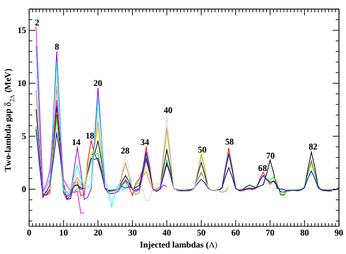

FIG. 1: Two lambda shell gap vs no. of injected lambdas for the multi-lambda hypernuclei under investigation.

Hypernuclei	Lambda magic number											
$^{16+n\Lambda}$O	2	8	-	-	-	-	-	-	-	-	-	-	-
$^{48+n\Lambda}$Ca	2	8	14	-	20	-	-	-	-	-	-	-	-
$^{58+n\Lambda}$Ni	2	8	14	-	20	28	34	-	-	-	-	-	-
$^{90+n\Lambda}$Zr	2	8	14	-	20	28	-	40	50	-	-	-	-
$^{124+n\Lambda}$Sn	2	8	14	-	20	28	34	40	50	-	-	-	-
$^{132+n\Lambda}$Sn	2	8	14	-	20	28	34	40	50	-	-	-	-
$^{208+n\Lambda}$Pb	2	8	-	18	20	28	34	40	50	58	-	70	82
$^{292+n\Lambda}$120	2	8	-	18	-	28	34	40	50	58	68	-	82
$^{304+n\Lambda}$120	2	8	-	18	20	28	34	40	50	58	68	70	82
$^{378+n\Lambda}$120	2	8	14	18	20	-	34	40	50	58	68	-	82

TABLE I: Lambda magic number predicted in various multihypernuclei under investigation.

References

[1] J. Schaffner, C. B. Dover, A. Gal, C. Greiner and H. Stocker, Phys. Rev. Lett. **71**, 1328 (2002).

[2] Y. Sugahara and H. Toki, Prog. Theor. Phys. **92**, 803 (1994).

[3] M. T. Win and K. Hagino, Phys. Rev. C **78**, 054311(2008).

[4] H. Shen, F. Yang and H. Toki, Prog. Theor. Phys. **115**, 325 (2006).

[5] D. Steppenbeck et. al., Nature **502**, 207 (2013).

[6] C. B. Dover and A. Gal, Prog. Part. Nucl. Phys. **12**, 171 (1984).