# Microscopic rotation particle coupling for triaxial proton emitters

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## Introduction

Study of triaxially deformed proton emitters provides us with rich information about the nuclear structure and decay properties at the extreme of the nuclear landscape, where many phenomenological assumptions may fail. The theoretical study of these nuclei can give us an opportunity to find some interesting features and phenomenon related to the nuclei beyond the drip lines, where the experimental data are scarce.

The simpler implementation of the rotation particle coupling [1, 2] is termed as the particle rotor model (PRM) where the rotor is assumed to be rigid. Often, a variable moment of inertia (VMI) is considered in order to take into account the nonrigid behavior of the core. In the microscopic rotation particle coupling the experimental core energies are utilized to obtain the matrix elements of the particle-plus-rotor (PR) system. This idea is based on coupled channel calculations for axially symmetric nuclei given in Refs. [2–4].

## Formalism

The rotor Hamiltonian is given by

$$H_{\mathrm{rot}} = \sum_{i=1,2,3} \frac{\hbar^2}{2\mathcal{I}_i} \vec{R}_i^2 \tag{1}$$

where $R$ is the angular momentum and $\mathcal{I}$ is the moment of inertia. In the PRM with VMI, $\mathcal{I}$ is given by

$$\mathcal{I}_i = \mathcal{I}_0(R)\, \sin^2\left(\gamma - \frac{2\pi i}{3}\right), \tag{2}$$

where $\mathcal{I}_0(R) = \mathcal{I}_0\sqrt{1 + bR(R+1)}$, and the parameter $b$ is tuned to fit the rotor energies. For the PR system $\mathcal{I}_0(R)$ is replaced by $\mathcal{I}_0(I)$, where $\vec{I} = \vec{R} + \vec{j}$ with $j$ denoting the angular momentum of the particle. This approximation in VMI leads to spurious lowering of some states with high $I$.

The total wave function for a given spin $(I, M)$ of the odd-even nucleus can be written in the laboratory system ($R-$representation) as [3, 4]

$$\Psi_{IM}(r,\omega) = \sum_{ljR\tau} \frac{\phi^I_{ljR\tau}(r)}{r} |ljR\tau, IM\rangle, \tag{3}$$

where $\frac{\phi^I_{ljR\tau}(r)}{r}$ and $|ljR\tau, IM\rangle$ are radial and angular part of the total wave function respectively. The calculation is usually done in $K$ representation. In $K$ representation the quantum number $\tau$ is the projection $K_R$ of $R$ on the rotor's 3-axis.
The $K$ and $R$ representations are related by an amplitude

$$\begin{aligned} A^{IK}_{j\Omega_p,RK_R} \;=\; & \sqrt{\frac{2R+1}{2I+1}} \langle j\Omega_p RK_R | IK\rangle \\ & \sqrt{1+\delta_{K_R,0}}\ , \end{aligned} \tag{4}$$

through the relation

$$|ljRK_R, IM\rangle = \sum_{K,\Omega_p}{}^{'} A^{IK}_{j\Omega_p,RK_R} |lj\Omega_p K, IM\rangle. \tag{5}$$

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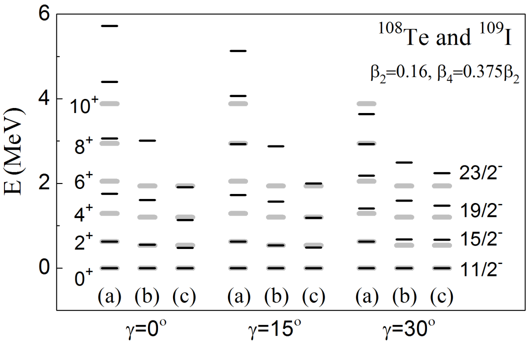

FIG. 1: Rotational spectra of $^{108}$Te and $^{109}$I calculated by using (a) parameter $b$ dependent VMI, here b=0.035 (b) PRM and (c) CMA at three $\gamma$ deformations. The grey lines correspond to the experimental spectra for $^{108}$ Te [5] and $^{109}$I [6].

The prime in the summation stands for the constraint that $(K - \Omega_p)$ must be an even integer. For the total Hamiltonian $H = H_{\rm av} + H_{\rm pair} + H_{\rm rot}$, the matrix element in $K$ representation can be written as

$$\langle q'K', IM|H|qK, IM\rangle = \epsilon_q \ \delta_{KK'}\delta_{qq'}$$
$$+ \sum_{lj} W^{KK'}_{j\Omega_p\Omega_{p'}} \times \int dr \ \phi^{IK'*}_{lj\Omega_{p'}}(r) \ \phi^{IK}_{lj\Omega_p}(r),$$

$\epsilon_q$ are the quasi particle energies. $H_{\rm rot}$ is not in diagonal form, so the matrix elements of the rotor Hamiltonian are coupled to the single particle basis states by

$$W^{K'K}_{j\Omega_{p'}\Omega_p} = \langle lj\Omega_{p'}K', IM|H_{\rm rot}|lj\Omega_p K, IM\rangle. \tag{6}$$

This $W^{K'K}_{j\Omega_{p'}\Omega_p}$ is the coupling matrix and this method is called coupling matrix approach (CMA).

## Result and discussions

The deformation parameters $\beta_2 = 0.16$ (0.13), $\beta_4 = 0.06$ (0.06) and $\gamma = 10^\circ$ (0) are predicted for $^{109}$I ($^{108}$Te) according to macroscopic-microscopic calculations. The measured spectrum of $^{108}$Te substantially deviates from that of an axially symmetric rotor. Many previous calculations consider this spectrum to be vibrational. However, a large triaxiality also can lead to such a spectrum. In Fig. 1(a), the calculated and experimental spectrum are shown for $^{108}$Te at three values of deformation $\gamma = 0^\circ, 15^\circ$ and $30^\circ$. Our results with a large value of VMI parameter $b = 0.035$ along with a strong triaxiality at $\gamma = 30^\circ$ are in good agreement with the experimental energies. For $^{109}$I, the rotational spectrum is calculated with two approaches, viz., the PRM with VMI and the CMA. The corresponding results are shown in Fig. 1(b) and (c), respectively. Since CMA utilizes the experimental core energies in which triaxiality is accounted for, the results are not very sensitive to $\gamma$. The CMA suggests $\gamma \lesssim 15^\circ$ for a good fit with the negative parity band in $^{109}$I. Similar results are obtained for the positive parity band also. The proton emitting state is a positive parity one and our preliminary calculations suggest a 3/2+ state in contrast to previous works.

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