Preformation probability of α-particle using proximity potential

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Introduction

Heavy unstable elements undergo α-decay in order to get rid of its extra positive charges, and thus attain stability by reducing the Coulomb energy. In 1928, α decay was explained as a quantum tunnelling process. The α decay constant (λ) is expressed as the product of three terms,

$$\lambda = P_0 \nu_0 P \tag{1}$$

Here, P_0 is the preformation probability of the α-particle, ν_0 is the assault frequency of the α-particle, and P is the probability of penetration of the α particle through the potential barrier. Technically speaking, the preformation probability of the α particle is defined as the probability of formation of the α particle as a separate cluster inside the nucleus before the emission process. The exact calculation of P_0 within a microscopic model is extremely difficult because of complexities involved in treating a nuclear many body problem. In this work we present an indirect calculation of P_0 using experimental values of λ.

Formalism

The relation between α-decay constant (λ) and half-life ($T_{1/2}$) is given by,

$$T_{1/2} = \frac{ln(2)}{\lambda} \tag{2}$$

From Eqs. (1) and (2), P_0 can be determined if $T_{1/2}$, ν_0 and P are known. For calculating the barrier penetrability (P) the WKB approximation is used and is given by,

$$P = exp\Big(-\frac{2}{\hbar}\int_a^b \sqrt{2\mu(V-Q)}dr\Big) \tag{3}$$

The turning points a and b are determined from the equation, $V(a) = V(b) = Q$, where Q is the Q-value of the disintegration. The interaction potential (V) between the two nuclei can be written as the sum of nuclear, Coulomb and centrifugal potentials. Hence,

$$V = V_C(r) + V_N(r) + \frac{\hbar^2 l(l+1)}{2\mu r^2} \tag{4}$$

where, r is the distance between the centres of the two nuclei, ℓ is the angular momentum quantum number, and μ is the reduced mass of the system. For determining the assault frequency, ν_0, we use the quantum mechanical formula,

$$\nu_0 = \frac{1}{h}\left\{\frac{Q}{\big(1+\frac{4}{A}\big)} + \frac{h^2}{4\mu R^2}\right\} \tag{5}$$

where, R is the radius of the parent nucleus and is given by [1],

$$R = 1.28A^{1/3} + 0.8A^{-1/3} - 0.76 \quad fm \tag{6}$$

The expression inside the curly brackets is the total energy of the α-particle, and is given by the sum of the energies on account of the Q-value, and the presence of the α-particle inside the potential of the parent nucleus. For the nuclear potential, the potential Prox 76 is used in which the constants γ_0 (=1.460734) and k_s (=4.0) of the original proximity potential (Prox 77) are slightly modified [2].

Results and discussion

Through the above outlined method we proceed to determine P_0 for all known even-even α-emitters. The experimental half lives and the Q values are taken from the latest data of Brookhaven National Laboratory, New York, USA [3].

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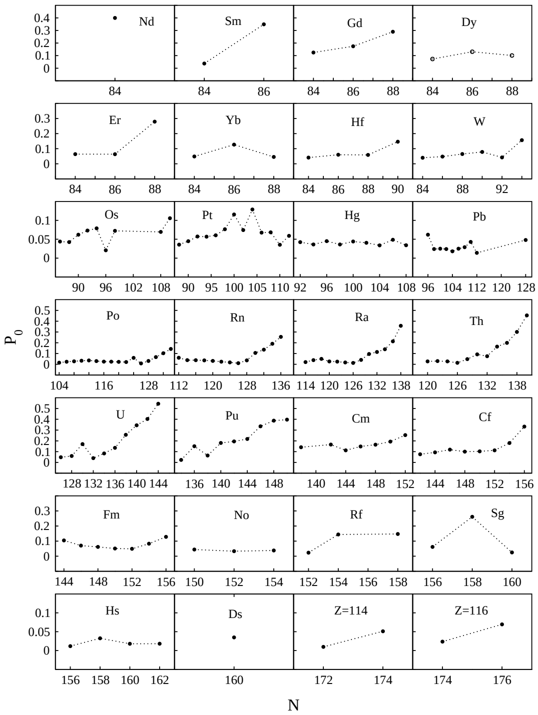

FIG. 1: Preformation probability (P_0) versus neutron number (N) for different isotopes of elements Nd, Sm, Gd, Dy, Er, Yb, Hf, W, Os, Pt, Hg, Pb, Po, Rn, Ra, Th, U, Pu, Cm, Cf, Fm, No, Rf, Sg, Hs, Ds, Z=114 and Z=116.

The calculated values of P_0 are shown in Fig. 1 as a function of the neutron number N for the different elements. The average value of P_0 is of the order of 0.1 which is in agreement with values obtained by other methods [4]. It is also observed that values of P_0 are in agreement with shell model as P_0 is minimum at the magic number N=126 for the elements Po, Rn, Ra and Th. Another minima at the semi-magic number N=152 is observed for the elements Fm, No and Rf. Another feature distinctly noted is the sharp rise in P_0 for N >126 for the elements Po, Rn, Ra, Th, U and Pu. This increase continues until there is saturation around N=152. Next, we attempt to parameterize the values of P_0,

$$logP_0 = a + b(Z - Z_1)(Z_2 - Z) + c(N - N_1)(N_2 - N) + dA \quad (7)$$

where, Z_1 and Z_2 are the proton magic numbers around Z ($Z_1 < Z \leq Z_2$), and N_1 and N_2 are the neutron magic numbers around N ($N_1 < N \leq N_2$). a, b, c and d are unknown constants whose values are fitted by minimising the standard deviation ($\sqrt{\sigma^2}$). The obtained values along with $\sqrt{\sigma^2}$ are shown in Table 1.

TABLE I: Values of the parameters a, b, c and d (Eq. 7)

Parameters	$50_{\text{Gd}}< Z \leq 82$ $82< N <126$	$82< Z$ $82< N \leq 126$	$50< Z \leq 82$ $126< N <152$
a	5.56927	5.9693	7.3927
b	0.003601	0.004101	0.004301
c	0.0046730	0.003773	0.003373
d	-0.062485	-0.060485	-0.060485
$\sqrt{\sigma^2}$	0.47840	0.74527	0.70291

Conclusion

In this work, α-particle preformation probability (P_0) is determined by using experimental values of half-lives for all known even-even α-emitters. The WKB approximation is employed, and values of penetrability (P) are calculated using Prox 76 potential. The values of P_0 obtained are found to be in agreement with the shell model as P_0 values obtained are minimum at the mabic number N=126, as well as at the semi-magic number N=152. Other potentials, like the relativistic mean field theory, the DDM3Y interaction, the Skyrme-Hartree-Fock mean field model, can also be used to study the values of P_0.

References

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