The Effect of Anisotropy of Elastic Scattering of Neutrons in the DPA Cross Sections of Light and Medium Mass Nuclei

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Introduction

The elastic scattering of neutrons plays an important role in displacing the lattice atoms and causing damage in the structural materials of a nuclear reactor. Its contribution to the displacement per atom (DPA) cross section of a material is significant over a large energy range. Though isotropic at low energies, the scattering of neutrons is anisotropic at high energies in the centre of mass frame. The scattering distribution becomes dominantly forward peaked with a decrease in the neutron interaction probability as energy increases. Hence, due to anisotropy of scattering an overall reduction is observed in the transferred energy to the primary knock-on atom (PKA) and DPA cross section of the material [2], compared to the condition of isotropic scattering.

Transfer of Energy to PKA

The probability for a neutron of energy E to emerge with energy E' after getting elastically scattered at centre of mass angle θ from the target is given by [4]

$$P(E \to E') = \begin{cases} \frac{4\pi\sigma_s(E,\theta)}{E(1-\alpha)\sigma_s(E)}, \alpha E < E' < E \\ 0, \quad otherwise \end{cases} \quad (1)$$

where $\sigma_s(E,\theta)$ is differential elastic cross section,

$\sigma_s(E)$ is total elastic cross section and

$\alpha = \left(\frac{A-1}{A+1}\right)^2$, A being the target mass in units of neutron mass. Then the average energy of the exit neutron is obtained from the equation

$$\overline{E}' = \int_{\alpha E}^{E} E' P(E \to E') dE' \quad (2)$$

Hence the average energy that is transferred to the PKA is given by

$$\overline{E}_R = E - \overline{E}' \quad (3)$$

For isotropic scattering, if we put

$\sigma_s(E,\theta) = \frac{\sigma_s(E)}{4\pi}$ in eq. (1), then we get

$$\overline{E}_R = E\frac{(1-\alpha)}{2}, \text{ for isotropic scattering} \quad (4)$$

For anisotropic scattering,

$$\sigma(E,\theta) = \frac{\sigma_s(E)}{2\pi}\sum_{l=0}^{NL}\frac{2l+1}{2}a_l(E_1)P_l(cos\theta)$$
$$= \frac{\sigma_s(E)}{2\pi} f(\mu,E) \quad (5)$$

with $\int_{-1}^{+1} f(\mu,E)d\mu = 1$, (6)

where $\mu = cos\theta$.

Then we get

$$\bar{E}_R = E\left[1 - \frac{1}{2}\int_{-1}^{+1}\left\{\sum_{l=0}^{NL}\frac{2l+1}{2}a_l(E)P_l(\mu)\right\}\{(1+\alpha) + (1-\alpha)\mu\}d\mu\right], \text{ for anisotropic scattering} \quad (7)$$

The values of $\overline{E}_R$ from eq. (4) and eq. (7) are calculated for natural carbon and ^{56}Fe. See figure 1. The data for scattering anisotropy, either Legendre polynomial expansion coefficients $a_l(E)$ or $f(\mu,E)$ as written in eq. (5) are obtained from ENDF/B-VII.1 [3].

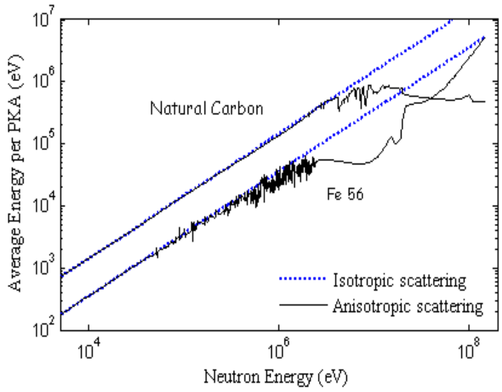

Fig. 1 the average energy transferred to a PKA of natural carbon and ^{56}Fe.

DPA Cross Section due to Elastic Scattering

The struck atom gets displaced from its lattice site only when the recoil energy E_R is greater than the atom displacement energy of the target element E_d. Furthermore, all of the kinetic energy of the PKA is not dissipated in producing atom displacements. A part of it goes into excitation of the atomic electrons. The fraction of PKA energy E_R that is spent in displacing the lattice atoms is termed as the damage energy T. This partitioning of energy E_R is given by the Lindhard energy partition theory. Based on that, analytical expressions have been obtained by Norgett, Robinson and Torrens (NRT) to evaluate the number of atom displacements that can occur from this T [1]. The number of atoms displaced by the dissipation of energy T is given by the NRT function

$$\nu[T\{E_R(\mu)\}] = \begin{cases} 0, T < E_d \\ 1, E_d \le T \le 2E_d \\ \frac{0.8T}{2E_d}, T > 2E_d \end{cases} \quad (8)$$

Hence, the DPA cross section due to elastic scattering is evaluated from equations (5), (8) and (9) as

$$\sigma_D^{el.}(E) = \int_{-1}^{+1} \nu[T\{E_R(\mu)\}]\,\sigma(E,\mu)d\mu$$

$$= \sigma(E)\int_{-1}^{+1} d\mu\, \nu[T\{E_R(\mu)\}] \sum_{l=0}^{NL} \frac{2l+1}{2} a_l(E)P_l(\mu) \quad (9)$$

For isotropic scattering

$$\sigma_D^{el.}(E) = \frac{\sigma(E)}{2}\int_{-1}^{+1} d\mu\, \nu[T\{E_R(\mu)\}] \quad (10),$$

with $l = 0$ *and* $a_0(E) = 1, P_0(\mu) = 1$.

$$= \sigma(E)\int_{-1}^{+1} d\mu\, \nu[T\{E_R(\mu)\}] \sum_{l=0}^{NL} \frac{2l+1}{2} a_l(E)P_l(\mu) \quad (11)$$

For isotropic scattering

$$\sigma_D^{el.}(E) = \frac{\sigma(E)}{2}\int_{-1}^{+1} d\mu\, \nu[T\{E_R(\mu)\}] \quad (12),$$

with $l = 0$ *and* $a_0(E) = 1, P_0(\mu) = 1$.

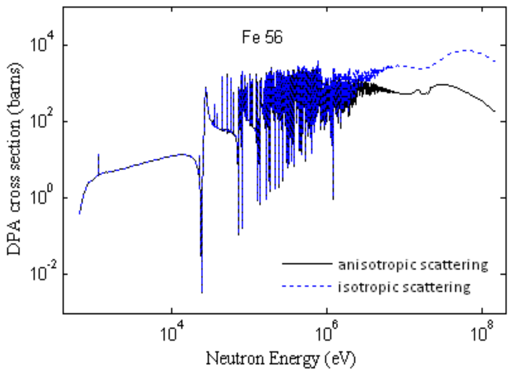

Fig. 2a DPA cross section due to elastic scattering of neutrons in ^{56}Fe.

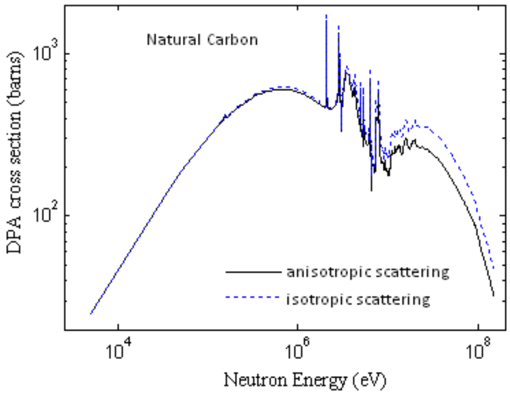

Fig. 2b DPA cross section due to elastic scattering of neutrons in natural carbon.

Discussions

The neutron scattering becomes anisotropic at about 45keV in ^{56}Fe and at 5keV in natural carbon. Around 2 MeV, DPA cross section due to elastic scattering is reduced by about 60% in ^{56}Fe and it is about 1.38% in carbon. Hence in a medium weight nucleus scattering becomes anisotropic at higher energy and reduction in DPA cross section is larger than that in a light nucleus. The anisotropy in case of inelastic scattering is very small compared to elastic scattering. These anisotropy effects have to be included while computing the total DPA cross sections of all structural materials.

References

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