# Angular momentum effects in the fusion of $^{28}$Si+$^{28}$Si system

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## Introduction

In the heavy ion fusion reactions the interaction potential plays an important role as it provides the characteristics like barrier height, barrier position and barrier width in the calculations of fusion cross section. This means different types of interaction potential gives different fusion cross sections or potential parameters are predicted w.r.t the experimental data. In the literature, number of formalism (e.g.[1, 2]) for the calculation of fusion cross sections assumes that the potential barrier position and width is independent of angular momentum ($\ell$). However, all the three potential characterstics are $\ell$-dependent and are used in the calculation the fusion cross section for a positive Q-value system, $^{28}$Si+$^{28}$Si (Q = 10.9 MeV) and is compared with the recently measured fusion cross section [3]. The nuclear proximity potential used is obtained in semiclassical extended Thomas-Fermi approach of Skyrme energy density formalism (SEDF) for Skyrme force SIV & the Coulomb, the centrifugal potentials are added directly to it to obtain total interaction potential. The fusion cross section is calculated using the partial wave analysis for the energies below and above the barrier. First, it is calculated by adding fusion cross section up to $\ell_{max} = 26\hbar$ as for $\ell > 26\hbar$ potential pocket vanishes and then cross section is summed up to that value of $\ell$, also called $\ell_{max}$, which fits the experimental data [3].

## Methodology

The fusion cross section in partial wave analysis is defined as,

$$\sigma_f(E_{cm}, \ell) = \frac{\pi\hbar^2}{2\mu E_{cm}} \sum_{\ell=0}^{\ell_{max}} (2\ell+1)P(E_{cm}, \ell) \quad (1)$$

where $P(E_{cm}, \ell)$ is the Hill-Wheeler approximation [4] for the penetration probability for a given partial wave and center of mass energy ($E_{cm}$),

$$P(E_{cm}, \ell) = \left[1 + \exp\left\{\frac{2\pi}{\hbar\omega_\ell}(V_{B\ell} - E_{cm})\right\}\right]^{-1} \quad (2)$$

where $V_{B\ell}$, $\hbar\omega_\ell$ are angular momentum dependent interaction barrier height and barrier width, respectively. The frequency $\omega_\ell$ related to interaction potential is calculated by applying a parabolic approximation at the top of interaction barrier using the equation given below,

$$V_T(R) = V_{B\ell} - \frac{1}{2}\mu\omega_\ell^2(R - R_{B\ell})^2 \quad (3)$$

where $R_{B\ell}$ is barrier position, $\mu$ is the reduced mass of interacting nuclei. The total interaction potential $V_T(R)$ is sum of nuclear proximity potential $V_N(R)$, Coulomb potential $V_C(R)$ and the centrifugal potential $V_\ell(R)$. The $V_N(R)$ is obtained using SEDF in semiclassical extended Thomas-Fermi approach, which in slab approximation (see Ref. [5]) becomes,

$$V_N(R) = 2\pi\bar{R}\int_{s_0}^{\infty}\left[H(\rho, \tau, \vec{J}) - \sum_{i=1}^{2} H_i(\rho_i, \tau_i, \vec{J_i})\right]dz$$

where $\bar{R} = R_{01}R_{02}/(R_{01}+R_{02})$ is the mean curvature radius, $H(\rho, \tau, \vec{J})$ is Skyrme Hamiltonian density [6] and $\rho(=\sum_i \rho_i)$, $\tau(=\sum_i \tau_i)$, $\vec{J}(=\sum_i \vec{J_i})$, are nuclear density, kinetic energy density and spin-orbit density respectively for composite system, $i = 1, 2$ for the two interacting nuclei. For the nuclear density we have used two parameter Thomas-Fermi density, which in slab approximation and after including temperature effects becomes,

$$\rho_i(z_i, T) = \rho_{0i}(T)\left[1 + \exp\left(\frac{z_i - R_{0i}(T)}{a_{0i}(T)}\right)\right]^{-1} \quad (4)$$

with $z_2 = R - z_1$ and $\rho_{0i}(T) = \frac{3A_i}{4\pi R_{0i}^3(T)}\left[1 + \frac{\pi^2 a_{0i}^2(T)}{R_{0i}^2(T)}\right]^{-1}$ is the temperature dependent central density, in which temperature is included through the temperature dependent central radii $R_{0i}(T)$ and surface thicknesses

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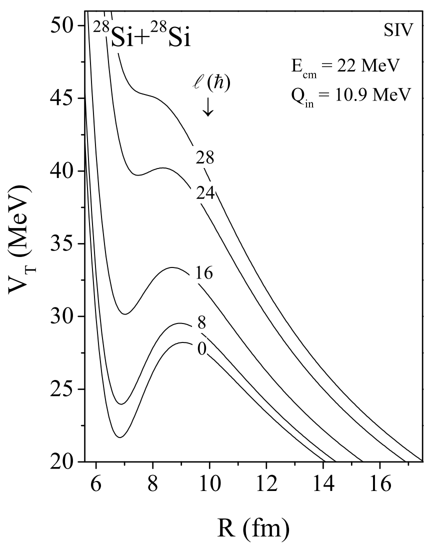


FIG. 1: (a) The total interaction potential as a function of inter-nuclear separation (R) at $E_{cm}$ = 22 MeV, for Skyrme force SIV.

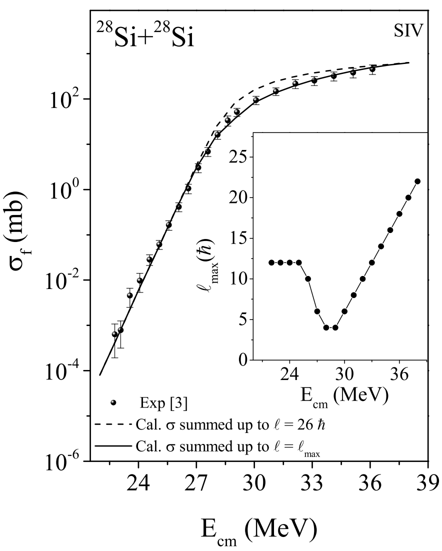


FIG. 2: The comparison of calculated fusion cross section as a function of $E_{cm}$ with the experimental data [3] for $\ell_{max}$ = 26ħ (dashed line) and at the different $\ell_{max}$ (solid line), which fits the data for Skyrme force SIV. The variation of required $\ell_{max}$ with $E_{cm}$ is shown in the inset figure.

$a_{0i}(T)$, (see Ref. [5]). The $E_{cm}$ is related to the temperature (T) as,

$$E^*_{CN} = E_{cm} + Q_{in} = \left(\frac{A}{9}\right) T^2 - T \qquad (5)$$

where $E^*_{CN}$ is compound nucleus excitation energy and $Q_{in}$ is the Q-value of incoming channel. The Coulomb potential and the centrifugal potential used are $V_C(= kZ_1Z_2/R)$ & $V_\ell(= \hbar^2\ell(\ell+1)/2\mu R^2)$, respectively.

## Calculations and results

Figure (1) shows the total interaction potential as function of inter-nuclear separation (R) at different angular momentum for $E_{cm}$ = 22 MeV. From the fig.(1), it is found that with increase in angular momentum, (i) $V_{B\ell}$ increases (ii) $R_{B\ell}$ shifts toward the lower values and (iii) barrier width ($\hbar\omega_\ell$) increases (iv) the depth of the pocket of the interaction potential decreases and vanishes at $\ell = 28\hbar$. This implies maximum angular momentum for which fusion cross section can be calculated for this system is $< 28\hbar$. The interaction potential characteristics are used in eqn.(2) to calculate the penetration probability and hence the fusion cross section using eqn.(1) at a given $E_{cm}$ for $\ell = 0$ to $26\hbar$. First, the fusion cross section is summed up $\ell = 26\hbar$ (shown with dashed line) and when compared with experimental data [3] (solid spheres) it is found that at low energies it is in nice agreement while at energies near and above barrier it is not able to reproduce the data (see fig.2). Therefore, to reproduce the data the calculated fusion cross section is summed upto that value of $\ell$, called $\ell_{max}$, which almost reproduces the observed data at a given $E_{cm}$, shown by the solid line in fig.(2). The required angular momentum at different $E_{cm}$ is shown in the inset of fig.(2).

The study reveals that at low energies i.e upto $E_{cm} \approx 25$ MeV, the fusion cross section is relatively small and become negligibly small above $\ell_{max} = 12\hbar$, thus further addition of angular momentum i.e upto $26\hbar$ have no effect on summed up fusion cross section. However, for energies above 25 MeV i.e near and above the barrier the cross section has relatively significant contribution from both below and above $\ell = 12\hbar$, hence we need different $\ell_{max}$ at different $E_{cm}$, lower than $26\hbar$.

## References


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