An estimate of Alpha decay half-life from the poles of S-matrix of an exactly solvable potential

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Introduction

α-decay of exotic nuclei has been time and again regained an extraordinary amount of attention for the calculation of half-lives and Q values. Furthermore, exactly solvable quantum mechanical models have proved its relevance in explaining fundamental quantum mechanical phenomenas. Our goal in this paper is to consider one such potential and study α-decay emission. We consider α-emission from a parent nucleus as a two body collision phenomena comprising of the ejected alpha particle and daughter nucleus exhibiting resonance states for which we find out the Q values and half-lives[1].

Formulation

The proposed potential [3] as a function of radial variable r is given by

$$V(r) = \begin{cases} V_0[S_1 + (S_2 - S_1)\rho_1] & if \quad r \le R_0 \\ V_0 S_2 \rho_2 & if \quad r \ge R_0 \end{cases} \quad (1)$$

$$\rho_n = \frac{1}{\cosh^2 \frac{R_0 - r}{d_n}}; n = 1, 2$$

where R_0 is the radial position having value; $R_0 = r_0(A_\alpha^{1/3} + A_D^{1/3}) + 2.73$, $r_0 = 0.97$ fm, $V_0 = 1MeV$. d_n accounts for the flatness of the barrier. In case of α+nucleus system, A_α and Z_α represents the mass and proton number of α particle. A_D and Z_D represent the mass and proton number of the daughter nucleus. Also S_1 and S_2 are the depth and height of the potential respectively having values;

$$S_1 = -78.75 + \frac{3 Z_\alpha Z_D e^2}{2R_c}$$

$$S_2 = \frac{Z_\alpha Z_D e^2}{R_0}(1 - \frac{a_g}{R_0})$$

where R_c is the Coulomb radius parameter; $R_c = r_c(A_\alpha^{1/3} + A_D^{1/3})$, $a_g = 1.6$ fm, $r_c = 1.2$ fm, $e^2 = 1.43996$ MeV fm. r_c and a_g are the distance parameters.

Using the potential given above, the Schrödinger equation is solved for the wave function which is expressed as a function of r as follows.

$$u_1(r) = A_1 Z_1^{i/2\kappa d_1} F(a_1, b_1, c_1, z_1) + B_1 Z_1^{-i/2\kappa d_1} F(a_1', b_1', c_1', z_1') \quad (2)$$

$$u_2(r) = A_2 Z_2^{i/2kd_2} F(a_2, b_2, c_2, z_2) + B_2 Z_2^{-i/2kd_2} F(a_2', b_2', c_2', z_2') \quad (3)$$

where;$\kappa^2 = k^2 - k_0^2 S_1$, $S = S2 - S1$, $k_0^2 = \frac{2m}{\hbar^2} V_0$, $k^2 = \frac{2m}{\hbar^2} E$

$$a_1 = \frac{1}{2}(\lambda_1 + i\kappa d_1), b_1 = \frac{1}{2}(1 - \lambda_1 + i\kappa d_1), \quad c_1 = 1 + i\kappa d_1$$

$$a_1' = \frac{1}{2}(\lambda_1 - i\kappa d_1), b_1' = \frac{1}{2}(1 - \lambda_1 - i\kappa d_1), \quad c_1' = 1 - i\kappa d_1$$

$$a_2 = \frac{1}{2}(\lambda_2 + ikd_2), b_2 = \frac{1}{2}(1 - \lambda_2 + ikd_2), \quad c_2 = 1 + ikd_2$$

$$a_2' = \frac{1}{2}(\lambda_2 - ikd_2), b_2' = \frac{1}{2}(1 - \lambda_2 - ikd_2), \quad c_2' = 1 - ikd_2$$

$$\lambda_1 = \frac{1}{2} - \frac{1}{2}[1 - (2\kappa d_1)^2 S]^{1/2}$$

$$\lambda_2 = \frac{1}{2} - \frac{1}{2}[1 - (2k_0 d_2)^2 S_2]^{1/2}$$

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Matching the wave functions and its derivatives with the regular Coulomb wave function F_l and irregular Coulomb wave function G_l, we obtain the S-matrix as

$$S_m = 2iC_l + 1 \quad (4)$$

where;

$$C_l = \frac{u_1 k F_l - u_1' F_l}{u_1'(G_l + iF_l) - u_1 k(G_l' + iF_l')}$$

We calculate the results of probability densities in two regions i.e. from $r = 0$ to $r = R_1, I_1 = \int_0^{R_1} |u_1|^2 dr$ and from $r = 0$ to $r = R_2$, with $R_2 > R_1, I_2 = \int_0^{R_2} |u|^2 dr$. The ratio of probability densities; $P = \frac{I_1}{I_2}$ as a function of center of mass energy $E_{c.m.}$ can be plotted [2]. The position of the peak represents the resonance energy or Q value. The same resonance energy can be obtained from the poles of S-matrix in the complex momentum plane. From a resonant pole of the S-matrix expressed by;

$$k_P = k_r - ik_i$$

We obtain the resonance energy (Q value)

$$E_P = \frac{\hbar^2}{2m}(k_r^2 - k_i^2) MeV \quad (5)$$

and Width

$$\gamma = \frac{\hbar^2}{2m} 4 k_r k_i MeV \quad (6)$$

The time of decay is related to the Width as

$$T_{1/2} = \frac{\hbar}{\gamma} \quad (7)$$

Results and Conclusion

We estimate the α-decay half-lives of several nuclei including light, heavy and super-heavy nuclei. The results of our calculated half-lives $T_{1/2}^{calt.}$ are compared with the corresponding experimental half-lives; $T_{1/2}^{expt.}$ in TABLE I. We find that our calculated results are coming out very close to the experimental ones.

TABLE I: Comparison between the experimental α-decay half-lives [4] and results of the present calculation. The value of the parameter d_1 is shown in parentheses under the respective nucleus keeping d_2 constant throughout i.e. $d_2 = 2$.

Nucleus	$Q_\alpha^{expt.}$(MeV)	$T_{1/2}^{expt.}(s)$	$T_{1/2}^{calt.}(s)$
$^{102}_{50}$Sn(4.90420)	4.290	8.0 x 10^{-5}	4.02 x 10^{-5}
$^{144}_{62}$Sm(4.94120)	3.271	2.2 x 10^{9}	2.35 x 10^{9}
$^{150}_{68}$Er(5.04180)	5.474	4.4 x 10^{-1}	2.89 x 10^{-1}
$^{174}_{80}$Hg(5.13240)	7.790	1.2 x 10^{-4}	2.23 x 10^{-4}
$^{214}_{82}$Pb(4.16359)	6.115	1.9 x 10^{2}	1.89 x 10^{2}
$^{234}_{90}$Th(3.71900)	4.270	1.4 x 10^{17}	5.59 x 10^{17}
$^{242}_{98}$Cf(3.99060)	8.377	1.3 x 10^{0}	1.92 x 10^{0}
$^{266}_{108}$Hs(4.18099)	11.117	2.1 x 10^{-4}	1.16 x 10^{-4}
$^{290}_{116}$Lv(5.40980)	11.810	1.4 x 10^{-3}	4.30 x 10^{-3}

Finally we conclude that by using an exactly solvable potential as mentioned above, we can find the poles of S-matrix and can efficiently estimate the $\alpha-$decay half-lives of various nuclei.

References

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