

## Status of $Z_c(4050)$ , $Y(4140)$ and $Y(4360)$ states as mixed P wave states of charmonium

Tanvi Bhavsar<sup>1,\*</sup>, Manan Shah<sup>2,†</sup> and P.C.Vinodkumar<sup>1‡</sup>

<sup>1</sup>Department of Physics, Sardar Patel University,  
Vallabh Vidyanagar-388 120, INDIA and

<sup>2</sup>P. D. Patel Institute of Applied Sciences, CHARUSAT, Changa-388 421, INDIA

### Introduction

During the past few years, many new charmonium states, such as X, Y, Z have been observed by BaBar [1, 2], Belle [3] and CLEO [4] experiments. The Belle collaboration has reported the discovery of  $Y(4360)$  [5],  $Z_c(4050)$  [6] and  $X(4350)$  [7]. The CDF Collaboration reported existence of a state called  $Y(4140)$  [8]. Present study is an attempt to identify some of these states as mixed P wave charmonia. Many of these states belongs to the P wave charmonia while quite a number of them qualified to be the mixed p-wave states [9].

### Theoretical Methodology

We assume here that quark and anti quark inside the mesonic bound state is independently confined by a potential [10],

$$V(r) = \frac{1}{2}(1 + \gamma_0)(\lambda r + V_0) \quad (1)$$

Where  $\lambda$  is the strength of the confinement part of the potential.  $V_0$  is a constant negative potential depth.

To get binding energy we have solved the two component (positive and negative energy) Dirac equation. Its solution can be written as [11],

$$\psi_{nlj}(r) = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} \quad (2)$$

where the positive and negative energy solu-

tions are given by [12],

$$\psi_A^{(+)}(\vec{r}) = N_{nlj} \begin{pmatrix} ig(r) \\ (\sigma \cdot \hat{r}) f(r) \\ r \end{pmatrix} \mathcal{Y}_{ljm}(\hat{r}) \quad (3)$$

$$\psi_B^{(-)}(\vec{r}) = N_{nlj} \begin{pmatrix} i(\sigma \cdot \hat{r}) f(r) \\ g(r) \\ r \end{pmatrix} (-1)^{j+m_j-l} \mathcal{Y}_{ljm}(\hat{r}) \quad (4)$$

Where  $N_{nlj}$  is the normalization constant.  $f(r)$  and  $g(r)$  are radial solutions which can be obtained numerically to get the energy eigen values. The parameters are fixed to get the ground state masses of  $c\bar{c}$ . Apart from the  $i$  dependent energy a residual OGEP is assumed in our earlier study of  $B, B_s$  meson [10] is also been incorporated. The predicted P-wave masses are listed in table:1.

TABLE I: P-wave mass spectrum for  $(c\bar{c})$  (in GeV).

nL	State	Present	Experimental [13]	[14]
1P	$1^3P_2$	3.53963	$(3556.20 \pm 0.0)$	3.54
	$1^3P_1$	3.53977	$(3510.66 \pm 0.07)$	3.50
	$1^3P_0$	3.50942	$(3414.75 \pm 0.31)$	3.44
	$1^1P_1$	3.40365	$(3525.42 \pm 0.29)$	3.51
2P	$2^3P_2$	3.86174	$(3927.2 \pm 2.6)$	4.02
	$2^3P_1$	3.86418	-	3.99
	$2^3P_0$	3.84709	-	3.94
3P	$2^1P_1$	3.76070	-	3.99
	$3^3P_2$	4.14130	-	-
	$3^3P_1$	4.14364	-	-
	$3^3P_0$	4.12729	-	-
4P	$3^1P_1$	4.05232	-	-
	$4^3P_2$	4.38978	-	-
	$4^3P_1$	4.39203	-	-
	$4^3P_0$	4.37619	-	-
	$4^1P_1$	4.30912	-	-

\*Electronic address: tanvibhavsar1992@yahoo.com

†Electronic address: mnshah09@gmail.com

‡Electronic address: p.c.vinodkumar@gmail.com

### Mixed P wave states

The mixed P wave states can be expressed as [15]

$$|\alpha\rangle = \sqrt{\frac{2}{3}}|{}^3P_1\rangle + \sqrt{\frac{1}{3}}|{}^1P_1\rangle \quad (5)$$

$$|\beta\rangle = -\sqrt{\frac{1}{3}}|{}^3P_1\rangle + \sqrt{\frac{2}{3}}|{}^1P_1\rangle \quad (6)$$

Where,  $|\alpha\rangle$   $|\beta\rangle$  are states having same parity. We can write the masses of these states in terms of the predicted masses of the pure P wave states ( ${}^3P_1$  and  ${}^1P_1$ ) as [15]:

$$M(|\alpha\rangle) = \frac{2}{3}M({}^3P_1) + \frac{1}{3}M({}^1P_1) \quad (7)$$

$$M(|\beta\rangle) = \frac{2}{3}M({}^1P_1) + \frac{1}{3}M({}^3P_1) \quad (8)$$

TABLE II: Mixing of p wave

Exp. state	Mixed state configuration	Present
$Z_c(4050)$	${}^3P_1$ and ${}^3P_1$	4.082
$Y(4140)$	${}^3P_1$ and ${}^3P_1$	4.1130
$Y(4360)$	${}^4P_1$ and ${}^4P_1$	4.3643

### Result and discussion

The predicted P-wave masses of charmonia are in very good agreement with experimental [13] results as given in Table I. We have also computed the masses of selected mixed P wave states and the results are given in table II. According to the results tabulated in table:2, we assign  $Y(4360)$  as a mixed state of  ${}^4P_1$  and  ${}^4P_1$  with a mixing angle of  $35.26^\circ$ ,  $Y(4140)$  as

a mixed state of  ${}^3P_1$  and  ${}^3P_1$  with a mixing angle of  $35.26^\circ$ . While  $Z_c(4050)$  is close to the mixed state  ${}^3P_1$  and  ${}^3P_1$  with a mixing angle  $54.74^\circ$ .

### Acknowledgments

We acknowledge the financial support from DST-SERB, India (research Project number: SERB/F/8749/2015-16)

### References

- [1] J. P. Lees et al.(BABAR Collaboration), Phys. Rev. D 86, 051102(R) (2012).
- [2] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. 98, 212001 (2007); J. P. Lees et al.
- [3] C. Z. Yuan et al. (Belle Collaboration), Phys. Rev. Lett. 99, 182004 (2007).
- [4] Q. He et al. (CLEO Collaboration), Phys. Rev. D 74, 091104(R) (2006).
- [5] Eur. Phys. J. C76 (2016) 387
- [6] WANG Zhi-Gang, Commun. Theor. Phys. 63 (2015) 466480.
- [7] Int.J.Mod.Phys.A30(2015)1550004.
- [8] C. P. Shen et al. (Belle Collaboration), Phys. Rev. Lett. 104,112004 (2010).
- [9] C. P. Shen et al. (Belle Collaboration),Phys. Rev. D 93, 112013 (2016).
- [10] Manan Shah, Bhavin Patel and P.C.Vinodkumar, Phys. Rev. D 93, 094028 (2016).
- [11] Manan Shah, Bhavin Patel and P.C.Vinodkumar, Phys. Rev. D 90, (2014) 014009.
- [12] Relativistic quantum mechanics wave equations by W. Greiner.
- [13] K.A.Olive et al.( Particle data group) chin. Phys.C, 38, 090001(2014).
- [14] J. Zeng, J. W. Van Orden and W. Roberts,Phys. Rev. D 52 5229 (1995), arXiv: hep-ph/9412269(1994)
- [15] Manan Shah, Bhavin Patel, P. C. Vinodkumar,Eur. Phys. J. C (2016) 76:36