Mass spectra of orbitally excited $c\bar{b}$ states in a non-relativistic quark model

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Introduction

The B_c state is the only bound system that consists of two heavy quarks of different flavours that offers a sound laboratory opportunity to observe both QCD and weak interaction. The ground state $B_c(1S)$ was first observed in the CDF and DO experiments (Tevatron) in two decay modes : $B_c \to J/\psi l\nu$ and $B_c \to J/\psi\pi$ [1, 2], also observed by LHC experiments in the various decay modes of $c\bar{b}$ states such as : $B_c \to J/\psi\pi$(LHCb, CMS, ATLAS), $B_c \to J/\psi\pi\pi\pi$ (LHCb and CMS), $B_c^+ \to \psi(2S)\pi^+$, $B_c^+ \to J/\psi D_s^+$ and $B_c^+ \to J/\psi D_s^{*+}$, $B_c^+ \to J/\psi K^+$ (LHCb) [2]. Recently the ATLAS collaboration at the LHC has observed radially excited $c\bar{b}$ state (i.e, $B_c(2S)$) through the decay channel $B_c^{\pm}(2S) \to B_c^{\pm}(1S)\pi^+\pi^-$ [3]. B_c mesons are predicted by the quark model to be members of the $J^P = 0^-$ pseudo scalar ground state multiplet [4]. The vector $B_c^*(1S)$ meson is the triplet state of $B_c(1S)$ which has not been observed by means of experiments to date.

Theory

In this work for the study of the B_c meson mass spectra, we have considered the following non relativistic Hamiltonian [5, 6],

$$H = K + V_{CONF}(\vec{r}_{ij}) + V_{OGEP}(\vec{r}_{ij}) \qquad (1)$$

where $V_{CONF}(\vec{r}_{ij})$ is the confinement potential

$$V_{CONF}(\vec{r}_{ij}) = -a_c r_{ij}\vec{\lambda}_i \cdot \vec{\lambda}_j \qquad (2)$$

K is the kinetic energy term

$$K = \left[\sum_{i=1}^{2} M_i + \frac{P_i^2}{2M_i}\right] - K_{CM} \qquad (3)$$

with M_i and P_i as the mass and momentum of the ith quark, respectively.

The central part of the two body potential due to OGEP [7] is given by

$$V_{OGEP}(\vec{r}_{ij}) = \frac{\alpha_s}{4}\vec{\lambda}_i \cdot \vec{\lambda}_j \left[\frac{1}{r_{ij}} - \frac{\pi}{M_i M_j}\left(1 + \frac{2}{3}\vec{\sigma}_i \cdot \vec{\sigma}_j\right)\delta(r_{ij})\right] \qquad (4)$$

where the first term represents the residual Coulomb energy and the second term is the chromo-magnetic interaction leading to the hyperfine splitting. σ_i is the Pauli spin operator and α_s is the quark-gluon coupling constant.

The non−central part of OGEP has two terms, namely the spin−orbit interaction $V_{OGEP}^{SO}(\vec{r})$ and tensor term $V_{OGEP}^{ten}(\vec{r})$. The spin-orbit interaction of OGEP is given by,

$$V_{OGEP}^{SO}(\vec{r}) = -\frac{\alpha_s}{4}\lambda_i \cdot \lambda_j \left[\frac{3}{8M_i M_j}\frac{1}{r^3}(\vec{r} \times \vec{p}) \cdot (\sigma_i + \sigma_j)\right] \qquad (5)$$

We use the following tensor term [8]

$$V_{OGEP}^{ten}(\vec{r}) = -\frac{\alpha_s}{4}\lambda_i \cdot \lambda_j \left[\frac{1}{4M_i M_j}\frac{1}{r^3}\right]\hat{S}_{ij} \qquad (6)$$

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where,

$$\hat{S}_{ij} = [3(\vec{\sigma}_i \cdot \hat{r})(\vec{\sigma}_j \cdot \hat{r}) - \vec{\sigma}_i \cdot \vec{\sigma}_j] \quad (7)$$

The tensor potential is a scalar which is obtained by contracting two second rank tensors.

TABLE I: **Masses of orbitally excited $c\bar{b}$ states (in MeV) for various n values.**

State $n\ ^{2S+1}L_J$	This work	Ref.[4]
$1\ ^3P_0$	6.679	6.700
$1P1$	6.709	6.730
$1P'$	6.746	6.736
$1\ ^3P_2$	6.757	6.747
$1\ ^3D_1$	7.000	7.012
$1D2$	7.006	7.012
$1D2'$	7.012	7.009
$1\ ^3D_3$	7.030	7.005
$2\ ^3P_0$	7.161	7.108
$2P1$	7.185	7.135
$2P1'$	7.208	7.142
$2\ ^3P_2$	7.215	7.153
$2\ ^3D_1$	7.424	
$2D2$	7.432	
$2D2'$	7.439	
$2\ ^3D_3$	7.449	
$3\ ^3P_0$	7.614	
$3P1$	7.630	
$3P1'$	7.643	
$3\ ^3P_2$	7.648	
$3\ ^3D_1$	7.845	
$3D2$	7.850	
$3D2'$	7.857	
$3\ ^3D_3$	7.863	

Results and Conclusions

The four parameters in our model are the mass of charm quark M_c, the mass of bottom quark M_b, the harmonic oscillator size parameter b and the quark-gluon coupling constant α_s. We use the following set of parameter values.
$M_c = 1.4$ GeV; $M_b = 4.645$ GeV; $\alpha_s = 0.3$; $a_c = 260.0$ MeV fm^{-1}; $b = 0.325$ fm.

For the case of a bound system of quark and anti-quark of unequal mass, charge conjugation parity is no longer a good quantum number so that the states with different total spins but with the same total angular momentum, such as the $^3P_1 -^1 P_1$ and $^3D_2 -^1 D_2$ pairs, can mix via the spin orbit interaction or some other mechanism. The B_c meson states with $J = L$ are linear combination of spin triplet $|^3L_J\rangle$ and spin singlet $|^1L_J\rangle$ states which we describe by the following mixing:

$$|nL'\rangle = |n\ ^1L_J\rangle \cos\theta_{nL} + |n\ ^3L_J\rangle \sin\theta_{nL} \quad (8)$$
$$|nL\rangle = -|n\ ^1L_J\rangle \sin\theta_{nL} + |n\ ^3L_J\rangle \cos\theta_{nL} \quad (9)$$

The values of the mixing angle for P states are $\theta_{1P} = 0.2°$, $\theta_{2P} = 0.10°$ and $\theta_{3P} = 0.05°$
The values of mixing angles for D states are $\theta_{1D} = 0.20°$ and $\theta_{2D} = 0.05°$.
The calculated masses of the $c\bar{b}$ states after diagonalization are listed in Table I.

Our prediction for masses of orbitally excited $c\bar{b}$ states are in good agreement with the other model calculations. Some of the states (i.e., 2^3P_0, 2P1,$2P1'$,2^3P_2) are 50-100 MeV heavier in our model.

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