Radiative decays of $c\bar{b}$ states in a non-relativistic quark model

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Introduction

The calculation of radiative (EM) transitions between the meson states can be performed from first principles in lattice QCD, but these calculation techniques are still in their development stage. At present, the potential model approaches provide the detailed predictions that can be compared to experimental results. In our model we consider the Magnetic dipole (M1) transitions and Electric dipole (E1) transitions of B_c meson:

Radiative Decays

Magnetic dipole (M1) transitions

Magnetic dipole (M1) transitions are those transitions in which the spin of the quarks is changed ($\Delta S = 1,\ \Delta L = 0$) and thus the initial and final states belong to the same orbital excitation but have different spins. Examples of such transitions are vector to pseudo scalar ($n\ ^3S_1 \to n'\ ^1S_0 + \gamma,\ n \geq n'$) and pseudo scalar to vector ($n\ ^1S_0 \to n'\ ^3S_1 + \gamma,\ n > n'$ meson decays.
The M1 partial decay width between S wave states is [1, 2]

$$\Gamma_{M1}(a \to b + \gamma) = \frac{16}{3}\alpha\mu_{eff}^2 k_0^3 (2J_b + 1) \times \left[\int_0^\infty R_{n_bL_b}(r) r^2 R_{n_aL_a}(r) dr\right]^2 \quad (1)$$

where $\int_0^\infty dr R_{n_bL_b}(r) r^2 R_{n_aL_a}(r)$ is the overlap integral for unit operator between the coordinate wave functions of the initial and the final meson states, m_c and m_b are the masses of the charm and bottom quarks and $\mu_{eff}^2 = \frac{m_bQ_c - m_cQ_{\bar{b}}}{4m_cm_b}$. S_a, S_b, L_a, J_a and J_b are the spin quantum number, orbital angular momentum and total angular momentum quantum numbers of initial and final meson states respectively.

Electric dipole (E1) transitions

Electric dipole (E1) transitions are those transitions in which the orbital quantum number is changed ($\Delta L = 1$, $\Delta S = 0$). Examples of such transitions are $n^3S_1 \to n'^3P_J\gamma(n > n')$ and $n^3P_J \to n'^3S_1\gamma(n \geq n')$. The strength of the electric dipole transitions is governed by the size of the radiator and the charges of the constituent quarks. The E1 partial decay width is given by [1],

$$\Gamma_{a\to b} = \frac{4\alpha}{9}\mu^2\left(\frac{Q_c}{m_c} - \frac{Q_{\bar{b}}}{m_{\bar{b}}}\right)^2 \frac{E_b(k_0)}{m_a}k_0^3$$
$$|\langle R_b|r|R_a\rangle|^2 \begin{cases} (2J+1)/3, & ^3S_1 \to ^3P_J \\ 1/3, & ^3P_J \to ^3S_1 \\ 1/3, & ^1P_1 \to ^1S_0 \\ 1, & ^1S_0 \to ^1P_1 \end{cases} \quad (2)$$

where k_0 is the energy of the emitted photon, $k_0 = m_a - m_b$ in non relativistic limit.
α is the fine structure constant. $Q_c = 2/3$ is the charge of the c quark and $Q_{\bar{b}} = 1/3$ is the charge of the $\bar{b}$ quark in units of $|e|$, μ is reduced mass $\mu = \frac{m_bm_c}{m_b+m_c}$ and
$\frac{E_b(k_0)}{m_a} = 1$ in non relativistic limit.

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$\langle R_b|r|R_a\rangle = \int_0^\infty r^3 R_b(r) R_a(r) dr$

is the radial overlap integral which has the dimension of length, with $R_{a,b}(r)$ being the normalized radial wave functions for the corresponding states.

Results and Conclusion

Transition	This work keV	Ref. [3] keV	Ref. [4] keV
$1^3P_0 \to 1^3S_1\gamma$	62.007	75.5	79.2
$1P1 \to 1^3S_1\gamma$	79.127	87.1	99.5
$1P1' \to 1^3S_1\gamma$	103.778	13.7	0.1
$1^3P_2 \to 1^3S_1\gamma$	112.798	122	112.6
$1P1 \to 1^1S_0\gamma$	37.055	18.4	0
$1P1' \to 1^1S_0\gamma$	47.261	147	56.4
$2^3S_1 \to 1^3P_0\gamma$	3.239	5.53	7.8
$2^3S_1 \to 1P1\gamma$	6.241	7.65	14.5
$2^3S_1 \to 1P1'\gamma$	3.283	0.74	0
$2^3S_1 \to 1^3P_2\gamma$	2.575	7.59	17.7
$2^1S_0 \to 1P1\gamma$	8.213	1.05	0
$2^1S_0 \to 1P1'\gamma$	2.443	4.40	5.2
$2^3P_0 \to 1^3S_1\gamma$	0		21.9
$2P1 \to 1^3S_1\gamma$	0		22.1
$2P1' \to 1^3S_1\gamma$	0		2.1
$2^3P_2 \to 1^3S_1\gamma$	0		25.8
$2P1 \to 1^1S_0\gamma$	0		
$2P1' \to 1^1S_0\gamma$	0		
$2^3P_0 \to 2^3S_1\gamma$	41.669	34.0	41.2
$2P1 \to 2^3S_1\gamma$	53.876	45.3	54.3
$2P1' \to 2^3S_1\gamma$	67.984	10.4	5.4
$2^3P_2 \to 2^3S_1\gamma$	72.711	75.3	73.8
$2P1 \to 2^1S_0\gamma$	20.795	13.8	
$2P1' \to 2^1S_0\gamma$	25.961	90.5	

TABLE I: **E1 transition rates of B_c meson.**

We have listed the possible $E1$ decay modes in table I and given the predictions for E1 decay widths. Most of the predictions for $E1$ transitions are in qualitative agreement. However, there are some differences in the predictions due to differences in phase space arising from different mass predictions and also from the wave function effects. For the transitions involving $P1$ and $P1'$ states which are mixtures of the spin singlet ${}^1P_1$ and spin triplet ${}^3P_1$ states, there exists huge difference between the different theoretical predictions.

The resulting M1 radiative transition rates of these states are presented in table II. In this table we give the calculated M1 decay widths for allowed transitions ($n^3S_1 \to n'^1S_0 + \gamma$, $n = n'$) and we compare the decay widths with other non relativistic quark models [4–6]. The hindered transitions are strongly suppressed in the non relativistic limit due to the orthogonality of the initial and final state wave functions. By adding relativistic effects to the wave function the hindered M1 transition rates can be enhanced.

Transition	Ref. [5] $\Gamma(keV)$	Ref. [6] $\Gamma(keV)$	Ref. [3] $\Gamma(keV)$	Ref.[4] $\Gamma(keV)$	This work $\Gamma(keV)$
$1\ {}^3S_1 \to 1^1S_0\gamma$	0.059	0.060	0.073	0.135	0.0581
$2\ {}^3S_1 \to 2^1S_0\gamma$	0.012	0.010	0.030	0.029	0.00173

TABLE II: **M1 transition rates for the B_c meson.**

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