Radiative decays of $c\bar{b}$ states in a relativistic quark model

Manjunath Bhat[1], Antony Prakash Monteiro[1,*] and K. B. Vijaya Kumar[2]
[1] *P.G Department of Physics, St Philomena College, Darbe, Puttur-574202,INDIA and*
[2] *Department Physics, Mangalore University, Mangalagangothri P.O., Mangalore - 574199, INDIA*

Introduction

We consider two types of radiative transitions of the B_c meson:

a) Electric dipole (E1) transitions are those transitions in which the orbital quantum number is changed ($\Delta L = 1$, $\Delta S = 0$). E1 transitions do not change quark spin. Examples of such transitions are $n^3S_1 \to n'^3P_J\gamma(n > n')$ and $n^3P_J \to n'^3S_1\gamma(n \geq n')$. The partial widths for electric dipole (E1) transitions between states ${}^{2S+1}L_{iJ_i}$ and ${}^{2S+1}L_{fJ_f}$ are given by

$$\Gamma_{a\to b} = \frac{4\alpha}{9}\mu^2\left(\frac{Q_c}{m_c} - \frac{Q_{\bar{b}}}{m_{\bar{b}}}\right)^2 \frac{E_b(k_0)}{m_a}k_0^3\,|\langle b|r|a\rangle|^2 \left\{\begin{array}{ll} (2J+1)/3, & {}^3S_1 \to {}^3P_J \\ 1/3, & {}^3P_J \to {}^3S_1 \\ 1/3, & {}^1P_1 \to {}^1S_0 \\ 1, & {}^1S_0 \to {}^1P_1 \end{array}\right. \quad (1)$$

where k_0 is the energy of the emitted photon,

$$k_0 = \frac{m_a^2 - m_b^2}{2m_a} \text{ in relativistic model.}$$

α is the fine structure constant. $Q_c = 2/3$ is the charge of the c quark and $Q_{\bar{b}} = 1/3$ is the charge of the $\bar{b}$ quark in units of $|e|$, μ is reduced mass $\mu = \frac{m_b m_c}{m_b + m_c}$ and $\frac{E_b(k_0)}{m_a} = 1$

$$\langle b|r|a\rangle = \int_0^\infty r^3 R_b(r)R_a(r)dr \quad (2)$$

is the radial overlap integral which has the dimension of length, with $R_{a,b}(r)$ being the normalized radial wave functions for the corresponding states.

b) Magnetic dipole (M1) transitions are those transitions in which the spin of the quarks is changed ($\Delta S = 1$, $\Delta L = 0$) and thus the initial and final states belong to the same orbital excitation but have different spins. Examples of such transitions are vector to pseudo scalar ($n\ {}^3S_1 \to n'\ {}^1S_0 + \gamma$, $n \geq n'$) and pseudo scalar to vector ($n\ {}^1S_0 \to n'\ {}^3S_1 + \gamma$, $n > n'$) meson decays.
The magnetic dipole amplitudes between S-wave states are independent of the potential model.

The M1 partial decay width between S wave states is [1, 2]

$$\Gamma_{M1}(a \to b + \gamma) = \frac{16}{3}\alpha\mu_{eff}^2 k_0^3(2J_b + 1) \times \left[\int_0^\infty R_{n_bL_b}(r)r^2 j_0(kr/2)R_{n_aL_a}(r)dr\right]^2 \quad (3)$$

where $\int_0^\infty dr R_{n_bL_b}(r)r^2 j_0(kr/2)R_{n_aL_a}(r)$ is the overlap integral for unit operator between the coordinate wave functions of the initial and the final meson states, $j_0(kr/2)$ is the spherical Bessel function, m_c and m_b are the masses of the charm and beauty quarks. S_a, S_b, L_a, J_a and J_b are the spin quantum number, orbital angular momentum and total angular momentum of initial and final meson states respectively.

Results and Conclusion

The possible $E1$ decay modes have been listed in Table I and the predictions for E1 decay widths are given. Most of the predictions for $E1$ transitions are in qualitative agreement. However, there are some differences in the predictions due to differences in phase space arising from different mass predictions and also from the wave function effects. For the transitions involving $P1$ and $P1'$ states which are mixtures of the spin singlet ${}^1P_1$ and spin triplet ${}^3P_1$ states, there exists huge difference between the different theoretical predictions. These may be due to the different

*Electronic address: aprakashmonteiro@gmail.com

$^3P_1 -^1 P_1$ mixing angles predicted by the different models. Wave function effects also appear in decays from radially excited states to ground state mesons such as $2\ ^3P_0 \rightarrow 1\ ^3S_1\gamma$. The overlap integral for these transitions in our model vanishes and hence we get decay width for these transitions zero.

Transition	This work keV	[3] keV
$1^3P_0 \rightarrow 1^3S_1\gamma$	42.384	75.5
$1P1 \rightarrow 1^3S_1\gamma$	83.879	87.1
$1P1' \rightarrow 1^3S_1\gamma$	121.143	13.7
$1^3P_2 \rightarrow 1^3S_1\gamma$	89.04	122
$1P1 \rightarrow 1^1S_0\gamma$	106.088	18.4
$1P1' \rightarrow 1^1S_0\gamma$	148.992	147
$2^3S_1 \rightarrow 1^3P_0\gamma$	3.635	5.53
$2^3S_1 \rightarrow 1P1\gamma$	0.384	7.65
$2^3S_1 \rightarrow 1P1'\gamma$	0.00751	0.74
$2^3S_1 \rightarrow 1^3P_2\gamma$	0.269	7.59
$2^1S_0 \rightarrow 1P1\gamma$	0.238	1.05
$2^1S_0 \rightarrow 1P1'\gamma$	0.00068	4.40
$2^3P_0 \rightarrow 1^3S_1\gamma$	0	
$2P1 \rightarrow 1^3S_1\gamma$	0	
$2P1' \rightarrow 1^3S_1\gamma$	0	
$2^3P_2 \rightarrow 1^3S_1\gamma$	0	
$2P1 \rightarrow 1^1S_0\gamma$	0	
$2P1' \rightarrow 1^1S_0\gamma$	0	
$2^3P_0 \rightarrow 2^3S_1\gamma$	0.422	34.0
$2P1 \rightarrow 2^3S_1\gamma$	104.751	45.3
$2P1' \rightarrow 2^3S_1\gamma$	158.896	10.4
$2^3P_2 \rightarrow 2^3S_1\gamma$	85.639	75.3
$2P1 \rightarrow 2^1S_0\gamma$	114.223	13.8
$2P1' \rightarrow 2^1S_0\gamma$	171.244	90.5

TABLE I: **E1 transition rates of B_c meson.**

Transition	$\Gamma(keV)$ Ref.[4]	$\Gamma(keV)$ This work
$1\ ^3S_1 \rightarrow 1^1S_0\gamma$	0.0189	0.0185
$2\ ^3S_1 \rightarrow 2^1S_0\gamma$	0.0037	0.0018
$2\ ^3S_1 \rightarrow 1^1S_0\gamma$	0.1357	0.193
$2\ ^3S_1 \rightarrow 1^3S_1\gamma$	0.1638	0.123

TABLE II: **M1 transition rates for the B_c meson.**

In order to calculate decay rates of hindered M1 transitions we need to include relativistic corrections. There are three main types of corrections: relativistic modification of the non relativistic wave functions, relativistic modification of the electromagnetic transition operator, and finite-size corrections.

We have calculated the M1 transition rates for $c\bar{b}$ meson states using equation (3). The resulting M1 radiative transition rates of these states are presented in table II. In this table we give calculated values for decay rates of M1 radiative transition in comparison with the other relativistic and non relativistic quark models. We see from these results that the relativistic effects play a very important role in determining the B_c meson M1 transition rates. The relativistic effects reduce the decay rates of allowed transitions and increase the rates of hindered transitions. The M1 transition rates calculated in our model agrees well with the values predicted by other theoretical models. It is clear from their calculations that the predicted decay rates for hindered transitions which are increased by relativistic effects almost by a factor of 3 and they are larger than the rates of allowed M1 transitions by an order of magnitude.

Acknowledgments

One of the authors (APM) is grateful to BRNS, DAE, India for granting the project and JRF (37(3)/14/21/2014BRNS).

References

[1] W. Kwong and J.L. Rosner, Phys. Rev. D **38**, 279 (1988).

[2] V. Novikov, L. Okun, M. Shifman, A. Vainshtein, M. Voloshin, and V. Zakharov, Phys. Rep. 41, 1 (1978).

[3] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 67, 014027(2003) `arXiv: hep-ph/0210381v2`.

[4] A. Abd El-Hady, J. R. Spence and J. P. Vary, Phys. Rev. D **71**, 034006(2005) `arXiv:hep-ph/0603139v1`.