Deuteron as a di-baryonic molecule

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Introduction
Deuteron have been widely studied and excepted as a bound state of proton and neutron [1, 2]. This molecular like interpretation and study have been attempted since last few decades. Various potentials like Yale, Paris, Argonne, Bonn etc..have been developed for the explanation of the deuteron properties [3]. According to the literature, it have been found that the one pion exchange play a trivial role for long range behaviour of the this bound sate as well as for the nuclear structure, but, it could not explain alone the properties of the interaction at short distance [1–3]. We have used the Hellmann potential [4] and one pion exchange potential [2] as interaction potential for short range and long range interaction, respectively, to calculate the binding energy. Using this interpretation and model we would like to study the di-baryonic states.

Theoretical Framework
We have solved the Schrödinger equation by using the virial theorem and hydrogen like trial wave function in the Ritz variational scheme. The effective Hamiltonian of the system is given by the equation [5–9]

\[ H = \sqrt{P^2 + m_1^2} + \sqrt{P^2 + m_2^2} + V(r_{12}) \]  (1)

Where the \( m_1 \) and \( m_2 \) are the masses of the constituent hadron and \( P \) is the relative momentum of the two constituents, whereas, \( V(r_{12}) \) is the net interaction potential under consideration, could be expressed as

\[ V(r_{12}) = V_h(r_{12}) + V_\pi(r_{12}) \]  (2)

Here, \( V_h(r_{12}) \) and \( V_\pi(r_{12}) \) are the Hellmann [4, 7, 9] and one pion exchange potential [2] respectively, given as

\[ V_h(r_{12}) = -\frac{k_{mol}}{r_{12}} + B \frac{e^{-\sigma r_{12}}}{r_{12}} \]  (3)

\[ V_\pi(r_{12}) = \frac{g^2}{4 \pi} \frac{m_\pi}{3} (\tau_i \cdot \tau_j) \left[ T_\pi(r)S_{12} + \left( Y_\pi(r) - \frac{4\pi}{m_\pi^2} \delta(r) \right) (\sigma_i \cdot \sigma_j) \right] \]  (4)

where \( T_\pi(r) \) and \( Y_\pi(r) \) could be given as

\[ T_\pi(r) = \left( 1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2} \right) e^{-m_\pi r} \]  (5)

\[ Y_\pi(r) = \frac{e^{-m_\pi r}}{m_\pi r} \]  (6)

Here, the constant \( k_{mol} \), \( B \) and \( C \) are the constant (appeared in the Hellmann potential). By using the hydogenic trial wave function and the virial theorem we have calculated the binding energy and mass of the deuteron.

\[ \langle K.E. \rangle = \frac{1}{2} \left\langle \frac{dV(r_{12})}{dr_{12}} \right\rangle \]  (7)

Result and discussion
We have analysed the Hellmann potential. The constant parameters of the Hellmann potential \( k_{mol} \) is the residual strength of the coulombic interaction whereas the constant \( B \) and \( C \) are the strength of the Yukawa like interaction, at short distance. In the fig.[1], the plot shows the change in the potential strength with change of \( B \) and \( C \) as once the coulombic interaction fixed. From the fig[1],
one can see that the value of the constant \( B \) increases the strength of the repulsive part of the potential, however, the constant \( C \) increases the attractive part of the curve [9].

The deuteron is excepted as a bound state of the proton and neutron. Our approach of the potential model in the variational scheme, we have calculated the binding energy and mass of the state. The constant of the Hellmann potential are fitted as, \( B = 6.5 \) and \( C = 0.235 \). The Fig. (2) depicted the nature of the potential for deuteron [9]. The binding energy from the calculation to be found

\[
BE_{(cal)} = 2.856 \text{ MeV} \quad \langle r^2 \rangle_{(cal.)} = 2.78 \text{ fm}
\]

\[
m_{(cal.)} = 1.874 \text{ GeV} \quad \langle r^2 \rangle_{(cal.)} = 2.14 \text{ fm}
\]

In summary, we have studied properties of the Hallmann potential with one pion exchange potential and extract the binding energy of the deuteron and test the model with model parameters. Indeed, we will apply this model to study the di-baryonic molecules.

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**References**


