# Effect of weak magnetic field on pion mass in vacuum

S. Biswas[1],* S. P. Adhya[2],† M. Mandal[3],‡ and P. K. Roy[4§]
[1] *Department of Physics, Rishi Bankim Chandra College, Naihati, Pin-743165, INDIA*
[2] *Experimental High Energy Physics and Application Group,*
*Veriable Energy Cyclotron Center, 1/A Bidhannagar, Kolkata-700 064, INDIA*
[3] *Department of Physics, Government General Degree College, Kalna-I,Burdwan-713405, INDIA and*
[4] *High Energy Nuclear and Particle Physics Division,*
*Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata-700 064, INDIA*

## Introduction

The study of the properties of mesons in presence of magnetic field has become an interesting topic of contemporary research. [1, 2]. Such investigations are important for the study of magnetars [3], color superconducting phases in the core of neutron stars [4, 5] etc. The other domain of applicability is RHIC.

In this work, we re-visit the pion propagation in presence of weak magnetic field [6]. The main ingradient of our calculation is the nucleon propagator in presence of magnetic field. Schwinger's proper time formalism [7] has been invoked to derive the nucleon propagator in presence of background magnetic field. We restricted our calculation to the one loop order for pion self energy using a phenomenological pion-nucleon ($\pi N$) interaction Lagrangian.

## Formalism

In the limit of weak magnetic field ($B$) the nucleon propagator can be written as [8]:

$$S(k) = S^{(0)}(k) + eB\, S^{(1)}(k) + (eB)^2\, S^{(2)}(k) + \mathcal{O}((eB)^3) \quad (1)$$

with

$$S^{(0)}(k) = \frac{\not k + m}{k^2 - m^2} \quad (2)$$

is the free fermionic propagator and

$$S^{(1)}(k) = \frac{i\gamma_1\gamma_2(\gamma.k_{||} + m)}{(k^2 - m^2)^2} \quad (3)$$

$$S^{(2)}(k) = \frac{-2k_\perp^2}{(k^2 - m^2)^4} \times \left[\not k + m - \frac{\gamma.k_\perp}{k_\perp^2}(k^2 - m^2)\right] \quad (4)$$

are the corrections to the propagator due to weak magnetic field. In deriving the above propagator we choose the $z$-axis as the direction of magnetic field. We define $k_{||}^2 = k_0^2 - k_z^2$, $k_\perp^2 = k_x^2 + k_y^2$ and $\sigma_3 = i\gamma_1\gamma_2$. The pion self energy of one loop order can be calculated from

$$\Pi(q) = -i\int \frac{d^4k}{(2\pi)^4} \times \mathrm{Tr}[\Gamma(q)S_a(k)\Gamma(-q)S_b(k+q)] \quad (5)$$

where the subscripts $a, b$ denote either $p$ (proton) or $n$ (neutron). $q$ and $k$ are pion and nucleon four momenta, respectively. The vertex factor of $\pi N$ interaction is denoted by $\Gamma(q)$. For simplicity we use $m_p = m_n = m$. In case of pseudoscalar interaction the vertex factor for neutral pion is $\Gamma(q) = -i\gamma_5 g_\pi$ and $g_\pi$ is replaced by $\sqrt{2}g_\pi$ for charged pions. $g_\pi$ is the pion-nucleon coupling constant. For pseudovector interaction $\Gamma(q) = (-i)\frac{g_\pi}{m}\gamma_5\not q$.

In the expression of pion self energy we keep the terms upto $(eB)^2$ and all the higher order terms have been neglected. It can be shown that the terms proportional to $eB$ in Eq.(5) have vanishing trace. Thus the self energy $\Pi(q)$ contains a term without magnetic field and the terms quadratic in magnetic field.

*Electronic address: `anjansubhra@gmail.com`
†Electronic address: `souvikadhya2007@gmail.com`
‡Electronic address: `mahatsab@gmail.com`
§Electronic address: `pradipk.roy@saha.ac.in`

The term without magnetic field is found divergent and we have applied dimentional regularization to isolate the divergent parts. Then proper renormalization cures the divergence and the self energy becomes finite. The terms proportional to $(eB)^2$ appear as the correction to the self energy due to weak magnetic field.

After calculating the pion self energy one can easily find the dressed pion propagator which is obtained by resumming the pion self-energy using the Dyson-Schwinger equation,

$$D(q) = D^0(q) + D^0(q)\Pi(q)D(q), \qquad (6)$$

where $D^0(q) = (q^2 - m_\pi^2 + i\epsilon)^{-1}$ is the bare propagator and $\Pi(q)$ is the pion self energy. The effective propagator can be written as,

$$D(q) = \frac{1}{[D^0(q)]^{-1} - \Pi(q)} \qquad (7)$$

Note that the pole of the effective propagator $D(q)$ determines the dispersion pion relation and effective mass of the pion is given by $m_\pi^* = \sqrt{m_\pi^2 + \Pi(\mathbf{q} = 0)}$.

## Results

In Fig.(1) and (2) we present variation of effective mass of pion as a function of magnetic field for pseudoscalar (PS) and pseudovector (PV) pion-nucleon interaction, respectively. It is observed that for PS interaction, effective mass of pion decreases with the increase of magnetic field strength, while it increases for PV interaction.

## Summary and Discussions

We have re-visited the modification of pion mass in presence of weak magnetic field. We have used Schwinger's proper time method for deriving fermion propagator in presence of background magnetic field and calulated pion self energy of one loop order. The effect of the external magnetic field appears as correction of order $(eB)^2$ over the vacuum contribution to the pion self energy. It is observed that the modification of pion mass due to magnetic field is marginal both for PS and PV interactions.

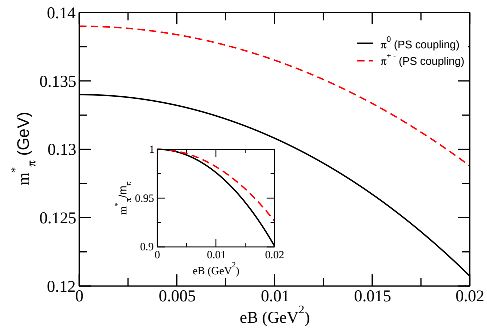


FIG. 1: Effective pion mass as a function of magnetic field (PS coupling).

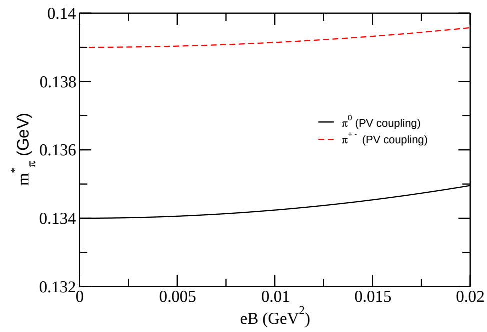


FIG. 2: Effective pion mass as a function of magnetic field (PV coupling).

## References


[1] K. Tuchin, Phys. Rev. C **88** 024910 (2013).

[2] A. Ayala, J. J. Cobos-Martnez, M. Loewe, M. E. Tejeda-Yeomans and R. Zamora, Phys. Rev. D **91** 016007 (2015).

[3] R. C. Duncan and C. Thompson, Astrophys. J. **392**, L9 (1992).

[4] E. V. Gorbar, Phys. Rev. D **62**, 014007 (2000).

[5] E. J. Ferrer, V. de la Incera, and C. Manuel, Phys. Rev. Lett. **95**, 152002 (2005), Nucl. Phys B **747** 88 (2006).

[6] G. Colucci, E. S. Fraga and A. Sedrakian, Phys. Lett. B **728** (2014) 19.

[7] J. Schwinger, Phys. Rev. **82**, 664 (1951).

[8] T. K. Chyi, C. W. Hwang, W. F. Kao, G. L. Lin, K. W. Ng and J. J. Tseng, Phys. Rev. D **62** (2000) 105014.