Qurakonia in Pöschl-Teller Potential using Supersymmetric (SUSY) Quantum Mechanics

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Introduction

In this introductory paper, we present the application of mathematical formalism of **SUSY QM** to give theoretical explanation to the experimental spectroscopic behaviors of quarkonia (mainly $b\bar{b}$). Till now there has been several attempts with varieties of inter-quark potentials for the understanding of its spectroscopic behaviors with reasonable accuracy. We have approximated inter-quark potential with Pöschl-Teller like potential. Using this non-relativistic model potential we have extracted values of potential parameters and then we have calculated theoretical values of the most well studied property of the quarkonium namely **di-leptonic decay widths** corresponding to various S- States.

The Pöschl-Teller Potential

The reason why we have selected Pöschl-Teller potential as an inter-quark potential is because it can accommodate several bound states decently and it is an exactly solvable potential and hence its wave functions and eigenvalues corresponding to various states can be calculated using SUSY QM formalism. The Pöschl-Teller potential is given as

$$V(r) = A^2 + (B^2 + A^2 + A\alpha)cosech^2(\alpha r) - B(2A + \alpha)\coth(\alpha r)cosech(\alpha r) \quad (1)$$

Here A, B and α are the potential parameters. Eigenvalues corresponding to this potential is given as [2]

$$E_n^0 = A^2 - (A - n\alpha)^2 \quad (2)$$

Here 'n' corresponds to number of states.
And wave functions corresponding to various states taking ħ=2m=1 is given as [2]

$$\psi_n^0(y) = (y-1)^{\frac{\lambda-s}{2}}(y+1)^{\frac{-(\lambda+s)}{2}} \times P_n^{\left(\lambda-s-\frac{1}{2},-\lambda-s-\frac{1}{2}\right)}(y) \quad (3)$$

Here $y = \cosh(\alpha r)$, $s = \frac{A}{\alpha}$, $\lambda = \frac{B}{\alpha}$ and $P_n^{(\alpha,\beta)}(y)$ is Jacobi polynomial.
Radial wave functions are written as

$$R_{n0} = \frac{\psi_n}{r} \quad (4)$$

Calculation of Potential Parameters

From equation (2) we can see that energy eigenvalues are dependent only on two potential parameters A and α. Experimental values [3] of energies of various S- state of bottomonium($b\bar{b}$) has been fitted with equation (2) with 95% confidence bounds to extract values of A and α. To calculate value of parameter 'B' we have used Van Royen-Weisskopf formula for di-leptonic decay width [1]. This formula relates di-leptonic decay width with radial wave functions and vector masses of mesons.

$$\Gamma_{ee} = \frac{4\alpha_e^2 e_Q^2}{M_v^2}|R_{nl}(r_0)|^2\left[1 - \frac{16}{3\pi}\alpha_s\right] \quad (5)$$

Here r_0 corresponds to the inter-quark separation (in fermi meter) for which radial wave function is attaining maxima, α_e is electromagnetic constant and α_s is strong coupling constant.
Using experimental value of di-leptonic decay width [3] of ground state of bottomonium, value of $|R_{00}(r_0)|^2$ is calculated and then we have taken several trial values of λ so that we can calculate that value of λ for which the experimental and theoretical value of $|R_{00}(r_0)|^2$ matches.
Calculated values of potential parameters are listed in table 1. Using these potential parameters we have calculated theoretical values of decay width which is listed in table 2.

Table 2: Comparison of experimental and theoretical values of decay width of bottomonium (for Pöschl-Teller potential).

State	Vector mass for nth state $M_v^n(b\bar{b})$	Peak value of R_{n0}^2	nth peak value of R_{n0}^2	avg peak value of R_{n0}^2	Experimental values of (Γ_{ee}) in KeV	Theoretical values of (Γ_{ee}) using peak value of R_{n0}^2 in KeV	Theoretical values of (Γ_{ee}) using nth peak value of R_{n0}^2 in KeV	Theoretical values of (Γ_{ee}) using avg peak value of R_{n0}^2 in KeV
1S	9.460	8.5465	8.5465	8.5465	**1.340 ± 0.018**	1.340	1.340	1.340
2S	10.023	7.3672	3.0023	5.1848	**0.612 ± 0.011**	1.029	0.419	0.724
3S	10.355	6.2444	1.5976	3.3340	**0.443 ± 0.008**	0.817	0.209	0.436
4S	10.579	4.8951	0.90698	2.1189	**0.272 ± 0.029**	0.614	0.114	0.266
5S	10.876	3.3717	0.49507	1.2270	**0.31 ± 0.07**	0.399	0.059	0.145

Table 1: Calculated values of potential parameters.

Potential parameters	Calculated values
A ($\sqrt{GeV}$)	1.261
α ($\sqrt{GeV}$)	0.2095
B ($\sqrt{GeV}$)	2.6068

We have calculated di leptonic decay width by taking peak value, nth peak value and average peak value of $|R_{n0}^2|$. By nth peak we intend to say the value of $|R_{n0}^2|$ when $|R_{n0}^2|$ rises to maxima for the nth time. This n should not be confused with number of states.

The plot of Pöschl-Teller potential with calculated values of parameters is shown in figure 1.

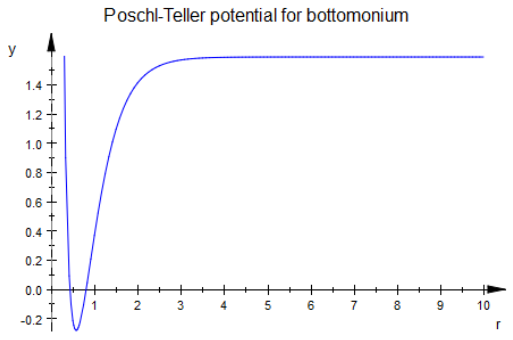

Figure 1: Pöschl-Teller potential with calculated value of potential parameters.

Conclusion and Discussion

From table 2 we conclude that Pöschl-Teller like potential can be approximated as inter-quark potential with good accuracy. Further, we can see that if we take average peak value of $|R_{n0}^2|$ instead of first peak or nth peak then we can explain spectroscopic properties of quarkonia in a much better way. On applying the same procedure to charmonium system, a similar conclusion was suggested. We believe that this procedure can be very useful to develop highly accurate inter-quark potential. The idea is to identify appropriate exactly solvable potentials. Then by calculating various potential parameters we can check the difference between experimental and theoretical values which can give us hints about necessary modifications required.

Refrences

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