Study of $B_s \to K^0 \nu \bar{\nu}$ decay in the light-cone quark model

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Introduction

The study of semileptonic decay processes of heavy-quark mesons is of great interest, since it is our main source that provides us an ideal field to study the mixing between different generations of quarks by extracting the most accurate values of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. It also provides insights on the hadron structure such as the hadron wave function and the hadron transverse momentum distribution [1]. The Bdecays have been the subject of many theoretical studies in the framework of Standard Model (SM) and the search of new physics. Bmesons are the only mesons containing quarks of the third generation and thus their decays provide a unique opportunity to measure the CKM matrix elements which describe the couplings of the third generation of the quarks to the lighter quarks. The currently operating B factories [2] make precision tests of the SM and beyond the SM ever more promising. Accurate analyses of the semileptonic and rare B decays are thus strongly demanded for such precision tests. With the upcoming chances that a numerous number of B_s mesons will be produced at hadron colliders, one might explore the rare B_s decays proceeding through $b \to d$ flavour changing neutral current (FCNC) transition, that open a window onto new physics beyond the SM. In order to make the accurate predictions within and beyond the SM, a reliable estimation of the hadronic form factors for the rare B decays is necessary. The information on the weak transition form factors is crucial for a proper extraction of the quark mixing parameters, to test the mechanism of CP violation in the SM

and for a search of new physics.

The aim of the present work is to calculate the hadronic form factor $f_+(q^2)$ for $B_s \rightarrow K^0 \nu \bar{\nu}$ decay within the framework of lightcone quark model (LCQM).

Framework

The LCQM is used to obtain the hadronic form factor $f_+(q^2)$. This model has an advantage of the equal light-cone time $(x^+ = x^0 + x^3)$ quantization and includes the important relativistic effects that are neglected in the traditional constituent quark model. In addition to this, the vacuum in this approach is nothing but emptiness. The light-cone wave functions are independent of the hadron momentum and thus are explicitly Lorentz invariant. A meson bound state consisting of a quark q_1 and an antiquark \bar{q}_2 with total momentum P and spin S is given by [3]

$$|M(P, S, S_z)\rangle = \int \frac{dp_1^+ d^2 \mathbf{p}_{1\perp}}{16\pi^3} \frac{dp_2^+ d^2 \mathbf{p}_{2\perp}}{16\pi^3} 16\pi^3 \\ \times \delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2) \sum_{\lambda_1, \lambda_2} \Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) \\ \times |q_1(p_1, \lambda_1) \bar{q}_2(p_2, \lambda_2)\rangle,$$
(1)

where p_1 and p_2 are the on-mass-shell light-front momenta,

$$\tilde{p} = (p^+, \mathbf{p}_\perp), \ \mathbf{p}_\perp = (p^1, \ p^2), \ p^- = \frac{m^2 + \mathbf{p}_\perp^2}{p^+}$$

The light-front momenta p_1 and p_2 in terms of light-cone variables are

$$p_1^+ = xP^+, \quad p_2^+ = (1-x)P^+,$$

$$\mathbf{p}_{1\perp} = x\mathbf{P}_{\perp} + \mathbf{k}_{\perp}, \quad \mathbf{p}_{2\perp} = (1-x)\mathbf{P}_{\perp} - \mathbf{k}_{\perp}.$$

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The momentum-space light-cone wave function Ψ^{SS_z} can be expressed as

$$\Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = R^{SS_z}_{\lambda_1\lambda_2}(x, \mathbf{k}_\perp) \ \phi(x, \mathbf{k}_\perp),$$

where $\phi(x, \mathbf{k}_{\perp})$ describes the momentum distribution of the constituents in the bound state and $R_{\lambda_1\lambda_2}^{SS_z}$ constructs a state of definite spin (S, S_z) out of the light-cone helicity (λ_1, λ_2) eigenstates.

Form factor $f_+(q^2)$ for $B_s \to K^0 \nu \bar{\nu}$ decay

At quark level, the decay $B_s \to K^0 \nu \bar{\nu}$ is explained through $b \to d$ transition. The form factor $f_+(q^2)$ can be obtained in $q^+ = 0$ frame with the "good" component of current, i.e. $\mu = +$, from the hadronic matrix elements given by [4]

$$\begin{split} \langle K^0 | \bar{d} \gamma^\mu b | B_s \rangle &= f_+(q^2) [(P_{B_s} + P_{K^0})^\mu \\ &- \frac{M_{B_s}^2 - M_{K^0}^2}{q^2} q^\mu] + f_0(q^2) \frac{M_{B_s}^2 - M_{K^0}^2}{q^2} q^\mu. \end{split}$$

The form factor $f_+(q^2)$ can be expressed in explicit form as [1]

$$f_{+}(q^{2}) = \int_{0}^{1} dx \int d^{2}\vec{k}_{\perp}\sqrt{\frac{\partial k'_{z}}{\partial x}}\sqrt{\frac{\partial k_{z}}{\partial x}}$$
$$\times \phi_{2}(x,\vec{k}'_{\perp})\phi_{1}(x,\vec{k}_{\perp})\frac{A_{1}A_{2}+\vec{k}_{\perp}\cdot\vec{k}'_{\perp}}{\sqrt{A_{1}^{2}+\vec{k}_{\perp}^{2}}\sqrt{A_{2}^{2}+\vec{k'}_{\perp}^{2}}}.$$
$$(2)$$

We shall perform our LCQM calculations in the $q^+ = 0$ frame, where $q^2 = q^+q^- - \vec{q}_{\perp}^2 = -\vec{q}_{\perp}^2 < 0$, and then analytically continue the form factor $f_+(q^2)$ from spacelike $(q^2 < 0)$ region to the timelike $(q^2 > 0)$ region by replacing \vec{q}_{\perp} to $i\vec{q}_{\perp}$ in the form factor [2]. The analytic solution of the form factor will be compared with the following parametric form

$$f(q^2) = \frac{f(0)}{1+a\ s+b\ s^2} \tag{3}$$

Calculations and Results

In the numerical calculations, we have used the constituent quark masses as $m_b = 4.8$ GeV, $m_d = 0.22$ GeV and $m_s = 0.37$ GeV and β parameters as $\beta_{B_s} = 0.56$ GeV and $\beta_{K^0} = 0.42$ GeV. In Fig. 1, we have shown the analytic solution for the form factor $f_+(s)$ (thick solid curve) for $0 \le q^2 \le (M_{B_s} - M_{K^0})^2$. We have also shown the results obtained from the parametric formula (dashed curve).



FIG. 1: Analytic solution of f_+ (thick solid curve) compared with the parametric result (dashed curve), with definition $s = \frac{q^2}{M_P^2}$.

As we can see from Fig. 1, the analytic solution given by Eq. 2 is well approximated by Eq. 3. The form factor at $q^2 = 0$ and the parameters *a* and *b* in Eq. 3 are 0.316, -1.424 and 0.533 respectively. These values can be compared with other theoretical results.

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