Relastic Results of Low-Lying Charmonium Masses Using Instanton Potential

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Introduction

Quantum chromodynamics (QCD) describes how quarks are bound inside hadrons by the strong force, mediated by the exchange of gluons.The theories of strong interactions, includes QCD in both the perturbative and non-perturbative regimes, LQCD, potential models and phenomenological models. The central part of the heavy-quark potential was first derived on the instanton liquid model for the QCD vacuum. The Wilson loop was averaged in the instanton ensemble to get the heavy-quark potential, which rises almost linearly as the relative distance between the quark and the antiquark increases, then it starts to get saturated. Though the instanton vacuum does not explain quark confinement, it will play a certain role in describing the characteristics of the quarkonia.
Charmonia are bound states of a charm and an anticharm quark ($c\bar{c}$), and represent an important testing ground for the properties of the strong interaction. Several quarkonium states have been observed after the discovery of the charmonium state j/ψ at BNL and SLAC.[1] The first observation of singlet ground state of charmonium η_c was done by Mark II and crystal Ball experiments in the radiative decays of j/ψ and ψ'. [1]

Theoretical Background

The inter-quark interactions include the linear confinement force and the one gluon exchange force. The potential energy term V which takes into account the interaction between the quark and the antiquark.

$$H = K + V \tag{1}$$

The Kinetic energy is given by

$$K = \sum_{i=1}^{2}\left(M_i + \frac{P_i^2}{2M_i}\right) - K_{CM} \tag{2}$$

K is the sum of kinetic energies including the rest mass minus the kinetic energy of the Center of mass of the total system[2].
Mass spectra of the quarkonia and their decays by solving explicitly the Schrdinger equation, combining the heavy-quark potential derived from the instanton vacuum[3] $V_{Q\bar{Q}}(r)$ with the confining $V_{conf}(r)$ and Coulomb potential $V_{coul}(r)$
Concerning the potential energy part V, we consider it as being composed of

$$V = V_{Q\bar{Q}}(r) + V_{conf}(r) + V_{coul}(r) \tag{3}$$

The static heay-quark potential is defined as the expectation value of the Wilson loop in a manifestly gauge-invariant manner. In this instanton liquid model for the QCD vacuum, we have two important parameters characterizing the dilute instanton liquid; the average size of the instanton $\rho = \frac{1}{3}$ fm, and average separation between instantons $\bar{R} = (N/V)^{-1}$ $\bar{R} \simeq 1fm$ where the instanton density is given as $N/V \simeq (200MeV)^4$ and number of colors N_C=3
The explicit form of the central potential from the instanton vacuum as[3]

$$V_C(r) = \frac{4\pi\bar{\rho}^3}{VN_c} I(\frac{r}{\bar{\rho}}) \tag{4}$$

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When the quark-antiquark distance is smaller than the instanton size, $r << \bar{\rho}(\frac{r}{\bar{\rho}} << 1)$, one can expand the dimensionless integral $I(\frac{r}{\bar{\rho}})$ with respect to $\frac{r}{\bar{\rho}}$. Which yields the central potential in the form of polynomial $V_C(r)$.[3] The static heavy-quark potential $V_{Q\bar{Q}}$ is defined as the expectation value of the Wilson loop in a manifestly gauge-invariant manner which yields the central potential in the form of a polynomial $V_C(r)$ and spin-dependent potentials $V_{SD}(r)$.

$$V_C(r) \simeq \frac{4\pi\bar{\rho}^3}{\bar{R}^4 N_c}\left(1.345\frac{r^2}{\bar{\rho}^2} - 0.501\frac{r^4}{\bar{\rho}^4}\right) \quad (5)$$

$$V_{SD} = V_{SS}(r)(\vec{S}_Q \cdot \vec{S}_{\bar{Q}}) + V_{LS}(r)(\vec{L}\cdot\vec{S}) + V_T(r)[3(\vec{S}_Q\cdot\hat{r})(\vec{S}_{\bar{Q}}\cdot\hat{r}) - (\vec{S}_Q\cdot\vec{S}_{\bar{Q}}] \quad (6)$$

Where

$$V_{SS} = \frac{1}{3{m_Q}^2}\nabla^2 V_C(r);$$

The expectaction values of $< \vec{S}_Q \cdot \vec{S}_{\bar{Q}} >$ depends on the total spin $(\vec{S})$ of the meson. Both spin orbit $V_{LS}(r)$ and tensor terms $V_T(r)$ affect the states with $L > 0$. The spin-spin term gives the spin-singlet-triplet splittings. The Confinement term represents the non pertubative effect of QCD. Which includes the spin-independent linear confinement term[4]

$$V_{conf} = -\left[\frac{3}{4}V_0 + \frac{3}{4}cr\right]F_1\cdot F_2 \quad (7)$$

Where c and V_0 are constants.F is related to the Gell-Mann matrix. Especially, we have $F_1\cdot F_2 = \frac{-4}{3}$ for the mesons. The coulomb - like (perturbative) one gluon exchange part of the potential is given by $V_{coul} = \frac{-4\alpha_s}{3r}$ with the strong coupling constant α_s .

Conclusions and Results

There are eight parameters in our model. These are the mass of charm quark m_c, the confinement strength c, the harmonic oscillator size parameter b, the coupling constant α_s. The constant parameters are ρ, $\bar{R}$, N_C and V_0.

$$m_c = 1.280\text{GeV}, \quad \text{c} = 40\text{GeV}^2,$$
$$b = 0.294 \text{ fm}, \qquad \rho = \frac{1}{3}\text{fm},$$
$$\bar{R} \simeq 1 \pm fm,\ V_0 = -250GeV\ , N_C = 3$$

We construct a 5X5 Hamiltonian matrix for low lying charmonium meson in the harmonic oscillator basis. In our calculation, the product of the quark-antiquark oscillator wave functions are expressed in terms of oscillator wave functions corresponding to the relative and centre of mass coordinates. The masses of the low lying charmonium mesons after diagonalization for successive values of n_{max}. We have presented preliminary results for the splittings of low-lying charmonium states. Table I shows our current estimates compared to the experimental values and Martin like potetnial model.

TABLE I: Masses of nS Low lying Charmonium States

$n^{2S+1}L_J$	Name	M_{exp}	Our Model	[5][a]
1^1S_0	$\eta_c(1S)$	2984	2981	2980
2^1S_0	$\eta_c(2S)$	3639	3762	3631
3^1S_0	$\eta_c(3S)$	3940	3762	3992
1^3S_1	J/ψ	3097	2969	3097
2^3S_1	$\psi(2S)$	3686	3554	3687
3^3S_1	$\psi(3S)$	4040	3797	4030

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