Z-expansion Analysis of the Proton Magnetic Radius

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Introduction

The proton is a fundamental constituent of matter. It is an extended object with finite size that can be inferred with some degree of accuracy from several measurements.

The electric radius can be extracted from electron-proton scattering experiments, ($r_E^p = 0.871 \pm 0.009$ fm) and Lamb shift in Muonic Hydrogen ($r_E^p = 0.84184 \pm 0.0006$ fm) ,($r_E^p = 0.84087 \pm 0.00039$ fm). The reason for the discrepancy between these values, often described as the "proton radius puzzle", is still unknown. In the literature there also exist several values of the proton magnetic radius, $r_M^p = 0.777 \pm 0.014$ fm (PDG 2014) or $r_M^p = 0.854 \pm 0.005$ fm or $r_M^p = 0.876 \pm 0.020$ fm. We will use the "z-expansion" method used in [1] to extract the magnetic radius of the proton from different scattering data [2]. We will also report the neutron magnetic radius extracted using the same technique.

Form factors and magnetic radius

The proton's extended structure can be probed by an electromagnetic current and is described by two form factors, known as Dirac and Pauli form factors. They are defined by F_1^N and F_2^N respectively in [2]

$$\langle N(p')|J_\mu^{\rm em}|N(p)\rangle = \bar{u}(p')[\gamma_\mu F_1^N(q^2) + \frac{i\sigma_{\mu\nu}}{2m_N} F_2^N(q^2)q^\nu]u(p) \quad (1)$$

where $q^2 = (p'-p)^2 = t$ and N stands for p or n. The Sachs electric (G_E) and magnetic form factor (G_M) are related to the Dirac-Pauli basis as [3]

$$G_E^N(t) = F_1^N(t) + \frac{t}{4m_N^2} F_2^N(t)\,, G_M^N(t) = F_1^N(t) + F_2^N(t)\,. \quad (2)$$

At $t = 0$, $G_E^p(0) = 1$, $G_E^n(0) = 0$, $G_M^p(0) = \mu_p \approx 2.793$, $G_M^n(0) = \mu_n \approx$ -1.913 [4].

The magnetic radius of the proton is defined as $r_M^p \equiv \sqrt{\langle r^2\rangle_M^p}$, where

$$\langle r^2\rangle_M^p = \frac{6}{G_M^p(0)} \frac{d}{dq^2} G_M^p(q^2)\bigg|_{q^2=0}. \quad (3)$$

Analyticity of form factors

The unknown functional behavior of the form factors makes it difficult to determine the no of parameters needed to fit experimental data. Therefore, our goal is to provide some constraints on the functional behavior of the form factors as shown in [1] by z-expansion method which is based upon the analytic properties of the form factor G_M^p as shown in Fig.1.

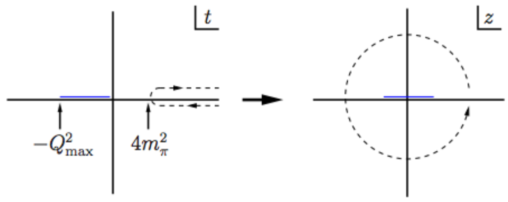

FIG. 1: Conformal mapping of the cut plane to the unit circle [1]

So we begin our approach by performing a conformal mapping of the domain of analyticity onto the unit circle by defining the variable

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z as

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}} \quad (4)$$

where for the present case $t_{\rm cut}= 4m_\pi^2$ and t_0 is a free parameter representing the point mapping onto $z = 0$. The form factors can be expanded in a power series in $z(q^2)$:

$$G(q^2) = \sum_{k=0}^{\infty} a_k\, z(q^2)^k \quad (5)$$

Coefficients

The analytic structure in the t-plane, illustrated in the Fig.1 implies the dispersion relation,

$$G(t) = \frac{1}{\pi}\int_{t_{\rm cut}}^{\infty} dt' \frac{{\rm Im}G(t' + i0)}{t' - t} \quad (6)$$

Parameterizing the unit circle by $z(t) = e^{i\theta}$ and solving (5) for t with changed limits we find

$$a_0 = \tfrac{1}{\pi}\int_0^{\pi} d\theta\, {\rm Re}\, G[t(\theta) + i0] = G(t_0) \ ,$$

$$a_k = \frac{2}{\pi}\int_{t_{\rm cut}}^{\infty} \frac{dt}{t - t_0}\sqrt{\frac{t_{\rm cut} - t_0}{t - t_{\rm cut}}} \times {\rm Im}G[t(\theta) + i0]\sin((k\theta(t)), k \geq 1 \quad (7)$$

Knowledge of this imaginary part of $G(t)$ helps us to put constraints on the coefficients a_k.

Proton magnetic radius extraction

For our data fitting we have used the z-expansion technique as described above,

$$G_M^p(q^2) = a_0 + a_1 z(q^2) + a_2 z^2(q^2) + ... \quad (8)$$

where $z(q^2) = z(q^2, t_{\rm cut}, t_0 = 0)$. We fit our data by minimizing a χ^2 function [1] ,

$$\chi^2 = \sum_i ({\rm data}_i - {\rm theory}_i)^2/(\sigma_i)^2, \quad (9)$$

where i ranges up to a given maximal value of Q^2, with $Q^2 = 0.1, 0.2, \ldots, 1.8$ GeV^2. We chose our default bounds on the coefficients to be $|a_k| \leq 10, 15$. The results shown here are for 8 parameters and 3 data sets (proton (P), neutron (N) and $\pi\pi$).

Q^2(GeV^2)	Bound	Proton magnetic radius r_M^p		
		P data	P+N data	P+N+$\pi\pi$ data
0.5	10	0.91	0.87	0.871
	15	0.92	0.87	0.873
1.0	10	0.91	0.88	0.874
	15	0.91	0.88	0.874

TABLE I: Proton magnetic radius

Q^2(GeV^2)	Bound	Neutron magnetic radius r_M^n		
		N data	N+P data	N+P+$\pi\pi$ data
0.5	10	0.74	0.89	0.89
	15	0.65	0.88	0.89
1.0	10	0.77	0.88	0.88
	15	0.74	0.89	0.88

TABLE II: Neutron magnetic radius

Conclusion

We extracted the magnetic radius of the proton as $r_M^p = 0.91^{+0.03}_{-0.06}\pm0.02$ fm,$0.87^{+0.04}_{-0.05}\pm$ 0.01 fm and $0.87^{+0.02}_{-0.02}$ fm for three data sets. We also extracted the radius of the neutron to be $r_M^n = 0.89 \pm 0.03$ fm which shows that within the error limit proton and neutron radius are consistent with each other.

References

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