D Wave mass spectra and triple gluon decay of Charmonia

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Introduction

The hadron spectroscopy is interesting due to the number of states observed experimentally [1]. The theoretical explanation of orbitally excited state of charmonium is challenging because many states may have possibilities of exotic states. As the mass of the charm quark is in the higher side we consider non-relativistic approach to determine the D wave mass spectra of charmonia [2]. And further we have calculated the triple gluon decay of Decay of the D wave state at zero seperation and at color compton radius. There are many methods to estimate the mass of a hadron, among which the phenomenological potential model is fairly reliable one, epically for heavier hadrons.

1. Theoretical Framework

A. Mass Spectra

The charmonium $c\bar{c}$ is made of c-quark and anti c-quark. We consider a Hamiltonian given by [2]

$$H = M + \frac{P^2}{2m} + V(r) \tag{1}$$

$M = m_Q + m_{\bar{Q}}$ and $m = \frac{m_Q m_{\bar{Q}}}{m_Q + m_{\bar{Q}}}$ Where m and $\bar{m}$ are the masses of quark and anti-quark, respectively, P is the relative momentum of each quark, and V (r) is the quark anti-quark potential. Though linear plus coulomb potential is a successful well-studied non-relativistic model for the heavy flavor sector, their predictions for decay widths are not satisfactory due to the improper value of the radial wave function at the origin compared to other models .

Recently, we have considered a general power potential with color coulomb term of the form

$$V(r) = -\frac{\alpha_c}{r} + Ar^{\nu} \tag{2}$$

Here, for the study of mesons, $\alpha_c = \frac{4}{3}\alpha_s$ Where, α_s is the strong running coupling constant, A is the potential parameter, and ν is in a general power such that the choice ν =1 corresponds to the coulomb plus linear potential.Choices of the power index in the range 0.7 to 1.3 have been explored. We have use the hydrogen like trail wave function of the form

$$R_{nl}(r) = \left(\frac{\mu^3(n-l-1)!}{2n(n+l)!}\right)^{1/2} (\mu r)^l e^{-\mu r/2} L^{2l+1}_{n-l-1}(\mu r) \tag{3}$$

Here μ is the vibrational parameter and L is the Laguerre polynomial. We employ the Ritz Vibrational scheme. We obtain the expectation value of the Hamiltonian as

$$H\psi = E\psi \tag{4}$$

we have taken the potential as shown below

$$V_{SD}(\mathbf{r}) = \left(\frac{\mathbf{L}\cdot\mathbf{S_Q}}{2m_Q^2} + \frac{\mathbf{L}\cdot\mathbf{s_{\bar{q}}}}{2m_{\bar{q}}^2}\right)\left(-\frac{dV(r)}{dr} + \frac{8}{3}\alpha_S\frac{1}{r^3}\right) + \frac{4}{3}\alpha_S\frac{1}{m_Q m_{\bar{q}}}\frac{\mathbf{L}\cdot\mathbf{S}}{r^3} + \frac{4}{3}\alpha_S\frac{2}{3m_Q m_{\bar{q}}}\mathbf{S_Q}\cdot\mathbf{s_{\bar{q}}}4\pi\delta(\mathbf{r}) + \frac{4}{3}\alpha_S\frac{1}{m_Q m_{\bar{q}}}(3(\mathbf{S_Q}\cdot\mathbf{n}) - (\mathbf{s_{\bar{q}}}\cdot\mathbf{n}) - \mathbf{S_Q}\cdot\mathbf{s_{\bar{q}}})\frac{1}{r^3}, \tag{5}$$

The first term of the equation takes into account the spin orbit interaction the second term takes into account the spin spin interaction and the third term takes into account the tensor interaction.

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TABLE I: Mass spectrum of $c\bar{c}$ meson (in GeV)

state	0.7	0.9	1	1.1	1.3	[3]
1^3D_3	3.574	3.678	3.723	3.766	3.850	3.813
1^3D_2	3.578	3.687	3.734	3.780	3.869	3.795
1^3D_1	3.575	3.684	3.732	3.778	3.868	3.783
1^1D_2	3.757	3.683	3.729	3.773	3.860	3.807
2^3D_3	3.833	4.032	4.125	4.216	4.396	4.220
2^3D_2	3.829	4.027	4.119	4.209	4.388	4.190
2^3D_1	3.819	4.012	4.101	4.188	4.359	4.105
2^1D_2	3.829	4.027	4.118	4.208	4.386	4.196

B. D wave decay into triple gluon

Two gluons cannot be emitted in $3D$ decays as the charge conjugation eigen value of a $3D$ state is odd. The resulting expression for the decay widths are [5] [6].
The decay is calculated both at origin and at color compton radius. $r(ccr) = \frac{2}{M}$ specifically for charmonia.

$$\Gamma(^3D_1 \to 3g) = \frac{760\alpha_S^3 R^2 \ln(4m_b r)}{81\pi m_b^6} \quad (6)$$

$$\Gamma(^3D_2 \to 3g) = \frac{10\alpha_S^3 R^2 \ln(4m_b r)}{9\pi m_b^6} \quad (7)$$

$$\Gamma(^3D_3 \to 3g) = \frac{40\alpha_S^3 R^2 \ln(4m_b r)}{9\pi m_b^6} \quad (8)$$

Where, R is the double derivative of the wave function at origin and at color compton radius and r is the average radius of the state.

2. Result and Discussions

The D wave masses of the chramonia is calculated by incorporating the non-relativistic phenomenological approach, using the hydrogen like trial wave function and potential term which incorporates spin orbit interaction, the spin spin interaction and the tensor interaction. The masses are calculated at different value of ν the masses are found to be in good agreement with the other reference and experimental value. Further the triple gluon of D wave of charmonia is calculated and it is observed that the result at color compton radius is better than the result at zero seperation and the decay width is compared with other reference as well.

TABLE II: ggg decay rates of $^3D_1$, $^3D_2$ and $^3D_3$ of $c\bar{c}$ meson at zero separation and at color compton radius (in keV)

$\Gamma_{ggg}(r=0)$ $\Gamma_{ggg}(r=ccr)$	0.7	0.9	1	1.1	1.3	[4]
$1^3D_1(r=0)$	736.826	1810.07	2591.51	3566.44	6249.39	160
$1^3D_1(r=ccr)$	58.601	83.971	91.990	96.299	93.080	
$1^3D_2(r=0)$	87.256	214.35	306.89	422.341	740.06	12
$1^3D_2(r=ccr)$	7.124	10.299	11.341	11.942	11.723	
$1^3D_3(r=0)$	349.023	857.401	1227.56	1689.36	2960.24	68
$1^3D_3(r=ccr)$	28.496	41.197	45.365	47.770	46.892	
$2^3D_1(r=0)$	61.459	161.541	235.645	338.396	617.431	
$2^3D_1(r=ccr)$	4.796	6.524	6.771	6.544	4.792	
$2^3D_2(r=0)$	7.278	19.130	27.905	40.073	73.117	
$2^3D_2(r=ccr)$	0.610	0.855	0.904	0.896	0.708	
$2^3D_3(r=0)$	29.112	76.520	111.621	160.293	292.467	
$2^3D_3(r=ccr)$	2.510	3.557	3.788	3.790	3.079	

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