One Dimensional Multifractal Study of Singly Charged Particles Produced in ^{32}S - AgBr Interactions at CERN SPS Energies

[1]Mir Hashim Rasool[*], [2]M. Ayaz Ahmad, [3]Ishfaq Majeed and [3]Shafiq Ahmad

[1]*Department of Physics, Islamic University of Science and Technology, Awantipora – 192122 - J&K INDIA*
[2]*Physics Department, University of Tabuk, Tabuk – 71451, SAUDI ARABIA*
[3]*Department of Physics, Aligarh Muslim University, Aligarh -202002, INDIA*
[*] *e-mail: hrasool23@gmail.com*

Introduction

R. C. Hwa [1] identified a new set of moment, called generalized moment, G_q, to study the multifractality and self-similarity in multiparticle production. However, if the multiplicity is low, the G_q moments are found to be dominated by statistical fluctuations. In order to suppress the statistical contribution, a modified form of G_q moments in terms of the step function was suggested by Hwa and Pan [1, 2], which can act as a filter for the low multiplicity events [1 - 3].

The properties of a multiply excited hadron can be explained by observing the multiplicity of particles and their distributions in variable phase space. We made an attempt to investigate some observations on multifractality of the multiplicity distributions of relativistic shower particles produced due to the interactions caused by 200 AGeV ^{32}S - beam with AgBr in the pseudorapidity phase space using the method of modified multifractal moments, G_q. The G_q moments have been calculated in terms of new-scaled variable $X(\eta)$. The variation of mass exponent function, τ_q is found to increase linearly with increasing order of moments. Physics output extracted from such an analysis could be helpful in revealing the dynamics of the particle production process. For this purpose, the total of 200 ^{32}S-AgBr events with mean multiplicity $\langle N_S \rangle$ = 94.53 ± 0.53 are investigated.

Experimental Technique

In this experiment two stacks of Ilford G5 nuclear emulsion plates exposed horizontally to a ^{32}S-beam at 200 AGeV obtained from Super Proton Synchrotron (SPS) at CERN have been utilized for data collection. The other relevant details about the present experiment can be found in our earlier publication [4-5].

Results and Discussions

The modified multifractal, G_q-moments is used to minimize the contribution of statistical fluctuations. In this approach, N single charged shower particles in a given X(η) interval, $\Delta X = X_{\max} - X_{\min}$ are distributed into M nonempty bins of width $\delta X = \Delta X / M$. A modified, G_q-moment is defined by the following relation:

$$G_q = \sum_{j=1}^{M} \left(n_j / N \right)^q \theta(n_j - q) \quad (1)$$

where q is a positive integer, n_j denotes the number of charged particles in the j^{th} bin and $N = \sum_{j=1}^{M} n_j$ is the total number of particles found in an event.

$$\theta(n_j - q) = \begin{cases} 1, & if\ n_j \geq q \\ 0, & if\ n_j < q \end{cases} \quad (2)$$

When averaged over all the events in a data sample in which the total number of events is N_{ev}, $\langle G_q \rangle$ is expressed as:

$$\langle G_q \rangle = \frac{1}{N_{ev}} \sum_{1}^{N_{ev}} G_q \quad (3)$$

A given rapidity distribution is said to exhibit self-similar behaviour and hence of fractal nature if $\langle G_q \rangle$ exhibits a power law behaviour [6] over a range of small δX in the following manner:

$$\langle G_q \rangle \propto M^{-\tau_q} \quad (4)$$

The exponent, τ_q called the fractal index, may be determined from the observed linear dependence of $ln\langle G_q\rangle$ on lnM over all windows using the relation:

$$\tau_q = \frac{\Delta \ln \langle G_q \rangle}{\Delta \ln (M)} \quad (5)$$

In order to see the existence of multifractality, the values of the modified multifractal moments, $\langle G_q\rangle$ for q = 2 - 6 have been calculated using Eqn. (3) for our data. The multifractal analysis of charged pions produced in ^{32}S-AgBr interactions at 200 AGeV has been investigated and are shown in Fig. 1 for different values of q. From these figures a linear dependence is observed in the values of $\ln \langle G_q \rangle$ with decreasing resolution of the bin width (i.e., increasing $\ln M$) for all values of q. The linear dependence of the modified multifractal moments gives an indication of self-similarity for the mechanism of particle production process and also reflects the fractal structure in multiparticle production. Thus the analysis of the present experimental work reveals an initial indication of fractal nature in multiparticle production system. Also we compare our experimental results with the results obtained from Lund Monte Carlo Code FRITIOF. It is observed from the figures that FRITIOF data also show a linear dependence on lnM similar to the experimental data. To calculate the statistical contribution to $\langle G_q\rangle$, 10000 events are generated using the uncorrelated Monte Carlo (MC-RAND) model in η space. The variation of $\ln\langle G_q\rangle$ with lnM for q = 2-6 for Monte Carlo events are also plotted in same figure. It has been found that the generated events show a little deviation from experimental values for the higher order of moments, q.

The linear behaviour is found to satisfy the power law dependence as described in Eqn. (5). The least square fitting of the data points in Fig. 1 have been done to find the values of the slopes i.e., mass exponent function, τ_q. The dependence of τ_q on the order of moments, q is shown in Fig. 2 for our experimental data along with the corresponding FRITIOF and Monte Carlo data. It is observed from the figure that the values of τ_q increases linearly with increasing order of moments for both experimental and FRITIOF data. The values of τ_q obtained by other workers [7, 8] are very much similar to our values obtained, indicating that the values of τ_q are nearly independent of collision energy and mass of projectile.

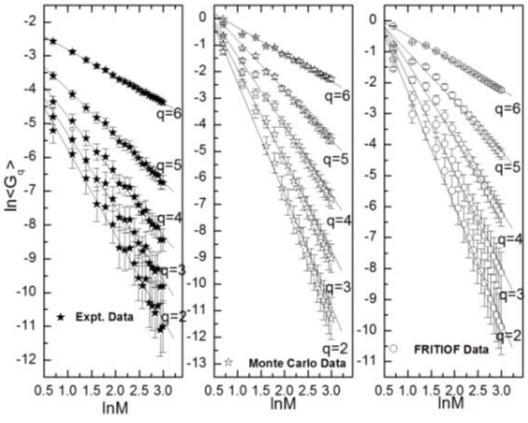

Fig. 1: Variation of $\ln\langle G_q\rangle$ as a function of lnM for experimental data along with MC-RAND and FRITIOF Data .

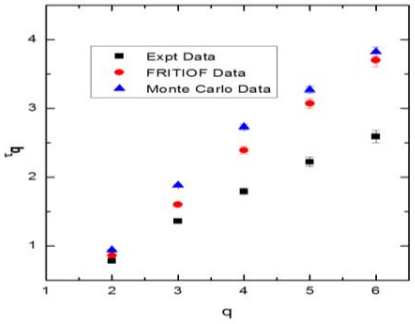

Fig. 2: Dependence of τ_q on order of the moments, q.

References

[1] R. C. Hwa Phys. Rev. D 41:1456 (1990)
[2] R. C. Hwa and J. Pan Phys. Rev. D 45:1476(1992)
[3] P. Carrruthers Int. J. Mod. Phys. A 4 : 5587(1989)
[4] M.H. Rasool et al. Jour. of Kor. Phys. Soc. 67:448 (2015).
[5] M. H. Rasool et al. Chaos Solitons and Fractals, 84:58 (2016)
[6] W. Florkowski and R. C. Hwa Phys. Rev. D 43:1548 (1991)
[7] G. Singh et al; Phys. Rev. C 50:2508 (1994)
[8] A. Bershadakii Phys. Rev. C 59:364 (1999)