Meson spectral functions in magnetized QGP

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Ongoing relativistic heavy ion collision experiments opportune us to study the properties of strongly interacting matter described by quantum chromodynamics (QCD) under extreme conditions. For high enough collision energies, a dense, hot and deconfined medium dubbed as quark gluon plasma (QGP) is created in the collision. Due to the fast motion of two oppositely directly ions an intense magnetic field is created in the early stage of non-central heavy ion collision. The energy scale associated with this magnetic field compares with the scale of QCD. To wit, the maximum value of the generated magnetic field could be as high as $eB \sim m_\pi^2$ at RHIC and $eB \sim 10m_\pi^2$ at LHC respectively. Here m_π stands for the pion mass and e is the charge of proton. An intense magnetic field modifies the QCD vacuum and entails a plethora of new phenomena - chiral magnetic effect, chiral vortical effect, modification of phase diagram to name a few.

Correlation functions of hadronic currents are useful tools to understand the intricate dynamics of QCD. The corresponding spectral densities carries information about in-medium hadron properties, off-equilibrium response and electromagnetic emissivity of the hot plasma. In this brief report, we discuss meson spectral functions in a magnetized quark gluon plasma. For similar calculation in the case of non-magnetized hot plasma see [1, 2]. We will restrict ourselves to $\mathcal{O}\left(\alpha_s^0\right)$ as far as the QCD corrections are concerned. Here α_s is the strong coupling constant. The electromagnetic interaction is included to all orders by construction. Furthermore, we take meson transverse momentum to be zero. This allows certain simplification in the calculation in that all the momentum integration can be calculated explicitly and the summation over discrete Landau levels is reduced to a bare minimum.

Formalism

The hadronic current is given by $J_H(\tau, \vec{x}) = \bar{q}(\tau, \vec{x})\Gamma_H q(\tau, \vec{x})$ where $\Gamma_H = 1, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \gamma_0$ for scalar (S), pseudoscalar (PS), vector (V), pseudo-vector (PV) and charge density correlator respectively. We assume a time independent background magnetic field along $+z$ direction. The background field breaks the isotropy of space and the finite temperature correlation function in an obvious notation is given by,

$$\mathcal{G}_\beta(\tau, x_\perp, z) = \langle J_H(\tau, x_\perp, z) J_H(0, 0_\perp, 0)\rangle_\beta$$
$$= T \sum\!\!\!\!\!\!\!\int_P e^{-i(\omega_n \tau - \vec{p}\cdot\vec{x})}\mathcal{G}_\beta(\omega_n, p_\perp, p_z) \quad (1)$$

Let us define the spectral density $\sigma_H(\omega, p_\perp, p_z)$ as, ,

$$\mathcal{G}_\beta(\omega_n, p_\perp, p_z) = \int_{-\infty}^{+\infty} dx \frac{\sigma_H(\omega, p_\perp, p_z)}{x - i\omega_n}$$
$$\Rightarrow \sigma_H(\omega) = \frac{1}{\pi}\Im\,\mathcal{G}_\beta(i\omega_n = \omega + i\epsilon) \quad (2)$$

The correlation function is obtained from the convolution of the fermion propagators in an external field which can be written as,

$$\widetilde{S_f}(x, x') = e^{\chi(x,x')} \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-x')} S_f(k) \ . \quad (3)$$

Here $S_f(k)$ is the translation and gauge invariant part of the fermion propagator in a background potential $A_\mu^{\rm ext}(x)$. The phase factor $\chi(x, x')$ is responsible for breaking of gauge and translation invariance. Explicit form of χ is irrelevant here. It drops out in a gauge invariant calculation.

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Let us assume a constant magnetic field along z direction. S_f can be decomposed as sum over projections over the discrete Landau levels,

$$
\begin{aligned}
iS_f(k) &= ie^{-\rho}\sum_{n=0}^{\infty}(-1)^n D_n\Delta_f^n \\
D_n\left(k_\parallel,k_\perp\right) &= 2\left(\not{k}_\parallel+m\right)\left(\mathcal{P}^- L_n(2\rho)\right. \\
&- \left.\mathcal{P}^+ L_n(2\rho)\right) - 4\not{k}_\perp L^1_{n-1}(2\rho) \\
\Delta_f^n\left(k_\parallel\right) &= \left(k_\parallel^2 - m^2 - 2n\left|q_f\mathcal{B}\right|\right)^{-1}. \quad (4)
\end{aligned}
$$

We decompose the four vectors into components $\parallel$ and $\perp$ to magnetic field, $a^\mu = a^\mu_\parallel + a^\mu_\perp$, where $a^\mu_\parallel = \left(a^0,0,0,a^3\right)$ and $a^\mu_\perp = \left(0,a^1,a^2,0\right)$. The metric tensor is $g^{\mu\nu} = g^{\mu\nu}_\parallel + g^{\mu\nu}_\perp$, where $g^{\mu\nu}_\parallel = \mathrm{diag}(1,0,0,-1)$ and $g^{\mu\nu}_\parallel = \mathrm{diag}(0,-1,-1,0)$. The scalar product is $a\cdot b = (a\cdot b)_\parallel + (a\cdot b)_\perp$ where $(a\cdot b)_\parallel = a^0b^0 - a^3b^3$ and $(a\cdot b)_\perp = -\left(a^1b^1 + a^2b^2\right)$. q_f is the charge of the fermion and $\rho = \mathbf{k}_\perp^2/|q_f\mathcal{B}|$. $P^\pm = \frac{1}{2}\left(1 \pm i\gamma^1\gamma^2\mathrm{sgn}\left(|q_f\mathcal{B}|\right)\right)$ are spin projection operators along the magnetic field direction. L_n^α are associated Laguerre polynomials. $L_n = 0$ if $n < 0$.

The one loop polarization tensor is calculated using (4) in various channels. For brevity, we quote here the result for spectral density in pseudoscalar channel which can be written as

$$
\begin{aligned}
\sigma_{ps}(\omega,p_z) &= \frac{|q_fB|}{4\pi^2}\sum_{r,f}\left(2-\delta_{r,0}\right)\frac{1}{\sqrt{1-\frac{4\epsilon_r^2}{s_\parallel}}} \\
&\times \left(1 - n_f\left(\omega_r^+\right) - n_f\left(\omega_r^-\right)\right). \quad (5)
\end{aligned}
$$

Here, $\epsilon_r^2 = m_f^2 + 2|q_fB|$, $\omega_r^\pm = \frac{1}{2}\left(\omega \pm p_z\sqrt{1-\frac{4\epsilon_r^2}{s_\parallel}}\right)$ and $s_\parallel = \omega^2 - p_z^2$. We considered here only light flavors $f = u,d$ in the chiral limit. It is to be noted that in the chiral limit scalar and pseudoscalar spectral functions are same even in presence of the magnetic field, $\sigma_{ps} = \sigma_s$.

In Fig.1, we display the results for pseudoscalar spectral function in the quark gluon plasma for a representative value of the magnetic field $eB = 6m_\pi^2$. For clarity we show here only the annihilation contribution. We find that the presence of magnetic field substantially modifies the shape of the spectral function. The sawtooth pattern in the spectrum is generated due the threshold singularity at each Landau level. We also show results for lowest Landau level approximation which can be read off from (5) by setting $r = 0$. Let us emphasize that LLL approximation is only valid for low frequency region and fails to reproduce the shape of the spectral function away from the threshold.

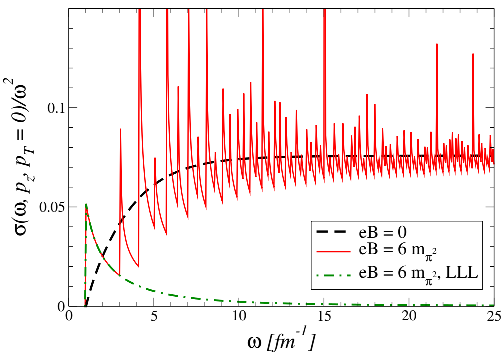

FIG. 1: Pseudoscalar spectral function in the chiral limit in a magnetized quark gluon plasma for $N_f = 2$. Only annihilation contribution is shown. $T = 200$ MeV and $p_z = T$.

Outlook

The present work should be extended in many ways. First of all, the transverse dynamics must be allowed and QCD correction should be included. It will be interesting to investigate in the present framework glueball and baryonic correlators. These are works in progress and will be reported elsewhere.

References

[1] W. Florkowski and B. L. Friman, Z. Phys. A **347**, 271 (1994).

[2] G. Aarts and J. M. Martinez Resco, Nucl. Phys. B **726**, 93 (2005).