Wavelet based fluctuation analysis in ^{16}O-AgBr interactions at different energies

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Introduction

In the context of fluctuation study of any physical observable it is important to correctly separate out the average behaviour or trend from the signal and to work with the actual fluctuation. There are several techniques in literature, like i) detrended fluctuation analysis [1,2], ii) wavelet transform modulus maxima (WTMM) [3], iii) wavelet transform [4, 5] based multiresolution analysis [6, 7], iv) Multifractal detrended fluctuation analysis (MFDFA) [2] etc. have been developed to eliminate the trend of the observable. They have been applied in different research areas to characterize the scaling behaviour and study the correlation [8-11]. The relative merits of MFDFA method over other techniques have been investigated [12]. Recently a new technique, which is akin to MFDFA approach, has been developed and used in [13]. In this approach a single wavelet of Daubechies family is used to appropriately separate out the trend from fluctuation. In [13] the authors carried out a systematic study with white noise using the wavelet based approach (using a number of wavelets from Daubechies family) and compared the results obtained from MFDFA method (with polynomial of different degrees). They found that for finite data set wavelet based approach is better suited. We have already carried out the MFDFA for the considered interactions. But due to the small length of Event-by-Event rapidity distribution of shower tracks again we investigate for better analysis and characterization of scaling properties.

Experiment

In the present analysis we have considered ^{16}O-AgBr interactions at 2.1 GeV/nucleon 60 GeV/nucleon. Small stacks of ILFORD G5 nuclear emulsion plates were exposed horizontally to ^{16}O beam during EMU-08 experiment [14] at CERN. More about the emulsion plates, flux density of incident beam and other aspects of the experiment can be found in reference [15]. Details regarding measurements like: instruments, scanning of plates, classification of tracks, event selection criteria and measurement of emission angle (θ) etc. can be found in our previous publications [16, 17]. In the present analysis we have considered only the shower tracks produced in the above stated interactions. From the measured emission angle (θ) pseudorapidity variable (η), has been calculated for further analysis using the equation ($\eta = -\ln\tan\frac{\theta}{2}$) for each of the shower tracks which are mainly pions.

Wavelet based fluctuation analysis method

This method is similar to MFDFA technique developed by Kantelhardt et al. [2] except in the process of detrending where we have used a Daubechies family wavelet instead of a polynomial fit. We have used Db4 wavelet for detrending. For a signal, $\{x_k, k=1, 2, 3\ldots, N\}$, of length N first the signal profile is calculated using eq^n (1) $y(j) = \sum_{k=1}^{j}[x_k - \langle x\rangle]$, where j runs from 1 to N ………..(1). Then profile *y(j)* has been divided into $N_s = int\,({}^{N}/_{s})$ numbers of non-overlapping segments of length *s*. Where $s=2^{(L-1)}W$ is the wavelet window size at a particular level (L) for the chosen wavelet. Here W is the number of filter coefficients of the discrete wavelet transform basis under consideration. Reconstruction of the low pass coefficient of the wavelet transformed series gives the local trend and subtracting those from original signal we obtain the actual fluctuations. The qth order fluctuation function is then obtained by squaring and averaging over all segments

$$F_q(s) = \left\{\frac{1}{2N_s}\sum_{v=1}^{2N_s}[F^2(v,s)]^{q/2}\right\}^{1/q} \quad \text{for } q \neq 0$$

$$= exp\left\{\frac{1}{4N_s}\sum_{v=1}^{2N_s}\ln F(v,s)\right\} \quad \text{for } q=0$$

…………(2)

Results and Discussions

Fluctuation function F_q (s) shows a power law behaviour. From the slop of log F_q (s) vs. log (s) we have calculated the generalized Hurst exponent $h_q(s)$ for different q. The variation of $h_q(s)$ with q has been presented in figure 1 for both of considered interactions.

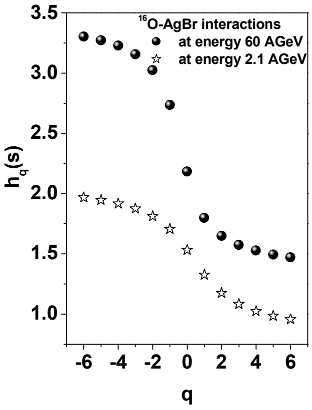

Figure 1. Variation of h_q with q

Decreasing values of $h_q(s)$ with q confirms that the pion production mechanism is multifractal in nature. The multifractal spectrum for both interactions has been given in figure 2.

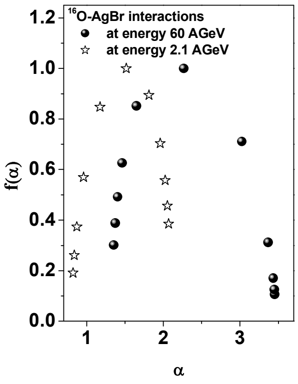

Figure 2. Multifractal specrtum

Multifractal width (1.2 & 2.1 for ^{16}O-AgBr interactions at 2.1 & 60 AGeV respectively) reveal that strength of multifractality increases with the projectile beam energy.

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