Multifractality in pp Collisions at LHC Energies

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The study of non-statistical fluctuations in relativistic nuclear collisions has attracted a great deal of attention of high energy physicists during the last three decades due to possibility of resolving information about the mechanism of multiparticle production in such collisions. Bialas and Peschanski[1] suggested the method of scaled factorial moments in order to find out the origin of non-statistical fluctuations in the multiplicity distributions of relativistic particles produced in these collisions. The phenomenon of intermittency reveals the self-similar behavior of multiplicity fluctuations in particle production at high energies. The concept of self-similarity is closely related to the fractal theory, which is a natural consequence of the cascading mechanism prevailing in the multiparticle production. The power-law behavior of the scaled factorial moments, indeed, implies the existence of some kind of fractal pattern[2] in the dynamics of the particles produced in their final state.

In this paper, efforts are made to apply the technique of multifractals[3] to investigate the multifractal structures in the multiparticle production using Monte Carlo events.

In order to examine the dependence of multifractal moments, G_q on η, a given pseudorapidity range $\Delta\eta = \eta_{max} - \eta_{min}$ is divided into M_0 bins of width $\delta\eta = \Delta\eta/M_0$. A multifractal moment is defined as;

$$G_q = \sum_{j=1}^{M} p_j^q \tag{1}$$

where, $p_j = n_j/n$, such that $n(= n_1+n_2......+ n_M)$. M denotes the number of nonempty bins. q is a real number and may has both positive or negative values. Once G_q is evaluated, its average over the sample is taken;

$$<G_q> = \frac{1}{N_{ev}} \sum_{1}^{N_{ev}} G_q \tag{2}$$

where, N_{ev} stands for the total number of events. If there is self-similarity in the production of particles, G_q moments can be written in the form of a power law;

$$<G_q> = (\delta\eta)^{\tau_q} \tag{3}$$

where, τ_q is mass exponents. The linear dependence of $< \ln G_q>$ on $\ln \eta$ over all the windows is related to

$$\tau_q = \lim_{\delta\eta \to \infty} (\Delta<G_q>/\Delta \ln \delta\eta) \tag{4}$$

The multifractal dimensions D_q and multifractal spectrum $f(\alpha_q)$ are related to the mass exponents τ_q as;

$$D_q = \frac{\tau_q}{q-1} \ \& \ f(\alpha_q) = q\alpha_q - \tau_q \tag{5}$$

where, $\alpha_q = d\tau_q/dq$ Here, D_0, D_1 and D_2 are known as fractal, information and correlation dimensions respectively.

In the present study, we have analyzed the Monte Carlo data(1 million events) generated by three different event generators AMPT-v1.21-v2.21, HIJING-1.35 and PYTHIA-8212 for pp collision at $\sqrt{S} = 0.9$, 2.76, 7.0, 13 TeV. Events with $N_{ch} \geq 10$(the number of charged particles) in the pseudorapidity range $\Delta\eta = -1 \longrightarrow +1$ are considered for the analysis.

The values of G_q moments for different pseudorapidity bin width have been calculated for pp collisions. In all three cases, the G_q moments with positive q values vary

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linearly over a wide range of $\delta\eta$ whereas, the moments having negative values of q, it saturate rather quickly as $\delta\eta$ decreases. This saturation effect could be due to the availabilty of a smaller number of particles as the bin width is reduced. The linear rise of the multifractal moments in η-phase space is an indication of self-similarity in the production mechanism. The slope τ_q of the linear region of the plots ln$\langle G_q \rangle$ versus -ln $\delta\eta$ for each data set are calculated using least square fit for first three data points. τ_q increases with increasing order q and exhibit almost similar trend of variation for all three generators and for a given q, τ_q is observed to be energy independent. The multifractal

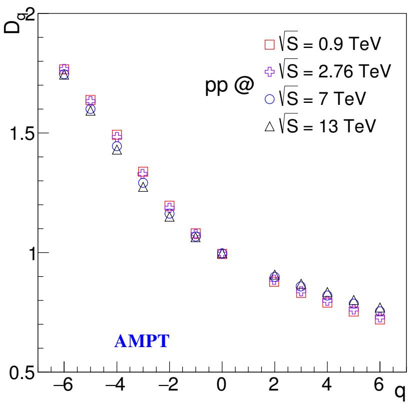

FIG. 1: The multifractal dimensions D_q as a function of q in η-space for pp collision.

moments D_q has been calculated for all data set and plotted against the order of the moments, q for all data set in Fig.1. We shown results here only for AMPT data set. For all cases D_q decreases with increasing q indicates that there exist multifractal behavior in multiparticle production in pp collisions.

Multifractal spectral function $f(\alpha_q)$ is calculated and plotted as the function of α_q in Fig.2 (for AMPT data). In each case, multifractal spectra, $f(\alpha_q)$ is represented by continuous curves. The figures show a distinct peak at $\alpha_q = \alpha_0(q = 0)$ and the solid line in each figure represents a common tangent at an angle of 45° at $\alpha_1 = f(\alpha_1)$. This is the consequence of a general property of the multifractals[2]. Concave downward trend of the spectra with a maximum at $q = 0$, $f(\alpha_0) = D_0 = 1$ gives an evidence for self similar cascade mechanism[4, 5].

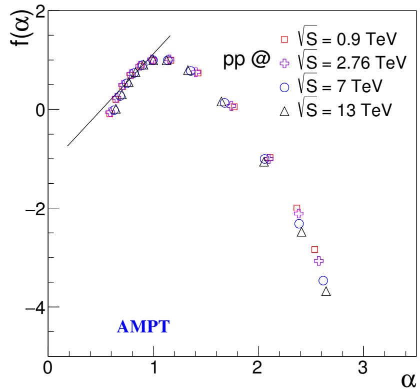

FIG. 2: The multifractal spectra $f(\alpha_q)$ as a function of α_q in η-space for pp collision.

Multifractility in multiplicity distributions of relativistic charged particles produced in pp collisions at LHC energies are examined by studying the variations of generalized dimensions, D_q and multifractal spectral function, $f(\alpha_q)$ with order of the moments, q. From the above study, we conclude that our results for the considered data are consistent with the intermittent behaviour and reveals the exsistence of multifractal structure in the data at LHC energies.

References

[1] Bialas A., Peschanski R., Nucl. Phys. B **273** , 703-718 (1986).

[2] Chiu C.B., Hwa R.C., Phys. Rev. D **43** , 100-103 (1991).

[3] Hwa R.C., Phys. Rev. D **41** , 1456-1462 (1990).

[4] Bialas A., Peschanski R., Phy. Lett. B **207** , 59-63 (1988).

[5] Ochs W., Wosiek J., Phy. Lett. B **214** , 617-620 (1988).