Taylor expansion in the study of neutron star crust-core transition density

T.R. Routray[1,*], X. Vi˜nas[2], D. N. Basu[3], S. P. Pattnaik[1,5], M. Centelles[2], L. M. Robledo[4], K. Madhuri[6] and B. Behera[1]
[1]School of Physics, Sambalpur University, Jyotivihar-768 019, INDIA
[2]Departament de F´ısica Qu`antica i Astrof´ısica and Institut de Ci`encies del Cosmos (ICCUB), Facultat de F´ısica, Universitat de Barcelona, Diagonal 645, E-08028Barcelona, SPAIN
[3]Variable Energy Cyclotron Center, 1/AF Bidhan Nagar, Kolkata, 700064, INDIA
[4]Departamento de F´ısica Te´orica (M´odulo15), Universidad Aut´onoma de Madrid, E-28049, SPAIN
[5]Department of Physics, Govt. Junior College, Sundargarh , Odisha, 770001, INDIA
[6]Govt. Women's Jr. College, Sambalpur-768001, Odisha, INDIA
* email: trr1@rediffmail.com

Introduction

The neutron star (NS) crust though forms a small fraction of the star mass and radius, it plays an important role in various observed astrophysical phenomena [1-3] related to neutron stars. In these studies the density and pressure at the crust-core transition are quantities of crucial relevance. The inner part of the NS crust is highly inhomogeneous that makes the formulation of the equation of state (EOS) in this region a formidable task and the value of the transition density, ρ_t, between the crust and the core still remains uncertain. Theoretically the transition density is ascertained by examining the onset of violation of the stability conditions of the homogeneous liquid core against small-amplitude density fluctuations, which indicates the formation of nuclear clusters. The resulting thermodynamical stability conditions contain the energy per particle in asymmetric nuclear matter (ANM) and its derivatives. The parabolic approximation (PA) resulting either from the Taylor expansion of energy or empirically constructed is often used for the energy expression of ANM. This PA gives results as good as the exact one in the domain of normal nuclear matter density with no exception found as yet. However, the contributions arising from the terms of higher order in asymmetry in the study relating to crust core transition region is a subject of current interest [4-8]. In this work we shall show that the Taylor expansion of energy density does not work in the region of crust-core transition.

Formalism

The thermodynamical stability conditions for a homogeneous matter in a given phase in the case of β -stable n+p+e+μ matter becomes,

$$V_{th} = 2\rho \frac{\partial e(\rho, Y_p)}{\partial \rho} + \rho^2 \frac{\partial^2 e(\rho, Y_p)}{\partial \rho^2} - \rho^2 \left(\frac{\partial^2 e(\rho, Y_p)}{\partial\rho\partial Y_p}\right)^2 \left(\frac{\partial^2 e(\rho, Y_p)}{\partial {Y_p}^2}\right)^{-1} > 0 \,, \quad ...(1)$$

where, $e(\rho, Y_p)$ is the energy per particle in ANM. This stability condition has been examined for the cases of exact and Taylor expanded energy density to obtain the transition density ρ_t using a simple finite range effective interaction (SEI) [8 and references therein],

$$v_{eff} = t_0(1 + x_0 P_\sigma)\delta(r) + \frac{t_3}{6}(1 + x_3 P_\sigma)\left(\frac{\rho}{1+b\rho}\right)^\gamma \delta(r) + (W + BP_\sigma - HP_\tau - MP_\sigma P_\tau)f(r) \quad (2)$$

Under the parabolic approximation,

$$e(\rho, Y_p) = e(\rho, Y_p = 1/2) + (1 - 2Y_p)^2 E_s(\rho), \quad ...(3)$$

where, $E_s(\rho)$ is the symmetry energy that can be obtained either from the 2nd order term of the Taylor expansion of energy density or by the popularly used empirical expression,

$$E_s(\rho) = [e^N(\rho, Yp = 0) - e(\rho, Yp = 1/2)] \quad ...(4)$$

with, $e^N(\rho, Yp = 0)$ and $e(\rho, Yp = 1/2)$ being energy per particle in pure neutron matter (PNM) and symmetric nuclear matter (SNM).

Results and Discussion

The transition density in NS is calculated from equation (1) for the exact and PA of e (ρ, Y_p) for the EOS of SEI in equation (2) corresponding to γ =1/2 that has incompressibility $K(\rho_0)$=237 MeV. The results are given in Table 1 along with the pressure and proton fraction. The results of Skyrme R_σ set are also given in the same table for comparison. It can be seen that the transition density predicted under PA is about 20% or more larger than the prediction of exact case. From this it can be concluded that the contributions coming from terms of higher order in the Taylor expansion of energy can't be neglected in the study of crust core transition. In order to examine this we have studied the quantity V_{th} given in equation (1) as a function asymmetry for a density in the region of the transition density. The results for the exact and PA cases for the EOS of SEI having γ=1/2 are shown in Figure 1 for density ρ=0.08 fm^{-3}. It can be seen that the PA agrees well with the exact one for asymmetry less than 0.5. However, with increase in asymmetry beyond this value, the curve for V_{th} corresponding to the exact expression of energy shows a very stiff rising behavior to which its PA counterpart can't compete. Since the proton fraction in the crust core transition region lies in the range around Y_p=0.025 where the difference in the behavior of V_{th} between the exact and PA becomes widely large, as can be seen from Fig.1, automatically explains the difference in the values of ρ_t obtained for the two cases. In order to see the influence of higher order terms of the Taylor series expansion, one needs to compute them for the SEI. However, it is a difficult task to compute the higher order terms of the Taylor series for a finite range interaction. But it can be done with relatively ease for Skyrme interaction due to analytically simpler energy expression. The calculation of V_{th} is done for the R_σ set upto 8th order and the results are shown in Figure 2 along with the exact result. It can be seen that in order to reproduce the behavior of the exact V_{th}, one need to include infinitely large higher order terms. This shows that in the region of crust-core transition the Taylor series breaks down and one should use the exact analytical expression for energy, where it is possible, to have correct prediction of the NS properties of the model.

Table 1. Transition density, pressure and proton fraction for the exact and PA of SEI and Skyrme R_σ.

SEI	EOS	ρ_t fm^{-3}	$P(\rho_t)$ MeV fm^{-3}	$Y_p(\rho_t)$
SEI γ=1/2	Exact	0.0788	0.430	0.0247
	PA	0.0953	0.742	0.0279
Sky R_σ	Exact	0.066	0.316	0.0143
	PA	0.093	0.898	0.0184

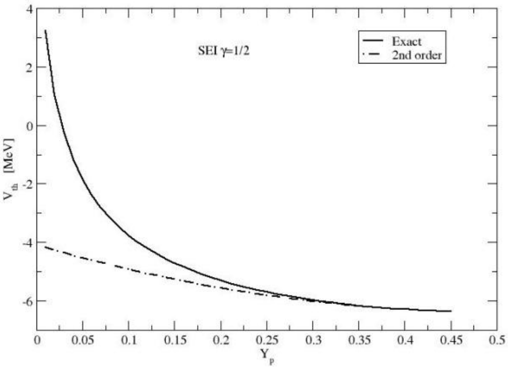

Figure 1. V_{th} as a function of the proton fraction Y_p at density ρ = 0.08 fm^{-3} for exact and 2nd order PA for the EOS of SEI.

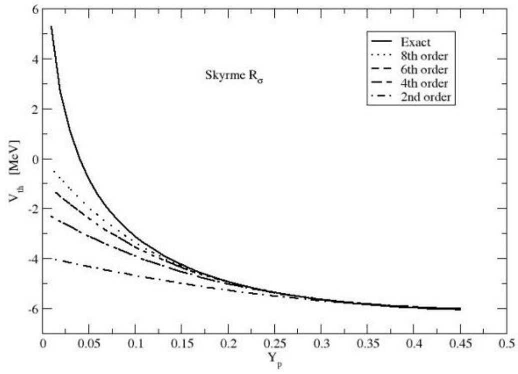

Figure 2. V_{th} as a function of the proton fraction Y_p at density ρ = 0.08 fm^{-3} for exact, 8th order, 6th order, 4th order and 2nd order Taylor expansion for the EOS of Skyrme interaction using R_σ set.

References

[1] Steiner A W, Prakash M, Lattimer J M and Ellis P J 2005 Phys. Rep. 410 325
[2] Newton W G, Gearheart M and Li B -A 2013 Astrophys J Suppl Ser 204 9
[3] Piekarewicz J, Fattoyev F J and Horowitz C J 2014 Phys. Rev. C 90 015803
[4] Xu J, Chen L W, Li B -A and Ma H -R 2009 Astrophys. J 697 1549
[5] Moustakidis Ch C 2012 Phys. Rev. C 86 015801
[6] Cai B -J and Chen L W 2012 Phys. Rev. C 85 024302
[7] Atta D and Basu D N 2014 Phys. Rev. C 90 035802
[8] Routray T R, Vi˜nas X, Basu D N, Pattnaik S P, Centelles M, Robledo L M and Behera B 2016 J. Phys G: Nucl.Part. Phys. (*in* press).