Non-cyclic geometric phases for neutrino oscillations in uniformly twisting magnetic fields

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The minimally extended standard model which includes massive neutrinos imparts non-trivial electromagnetic properties to the neutrinos which hold fundamental importance in the study of sterile neutrinos, deciding Dirac or Majorana nature of neutrinos, neutrino interaction cross sections etc [1]. The coupling of neutrino magnetic moments with the external magnetic fields leads to non trivial geometric phases of the neutrino eigenstates which influences the neutrino spin precession and may result in spin-flavor transitions [2]. In [2], the geometric phases were obtained for the case of adiabatic and cyclic evolution of the system. Here we extend the results for the more general case of non-adiabatic, non-cyclic evolution using kinematic approach of Mukunda-Simon [3]. In this approach, for a quantum state $\psi(s)$, s being a real parameter, belonging to a smooth open curve of unit vectors in Hilbert space, the gauge and reparametrization invariant geometric phase can be defined as

$$\phi_g[C] = arg\,\langle\psi(s_1)|\psi(s_2)\rangle - \Im\int_{s_1}^{s_2} ds\,\langle\psi(s)|\dot{\psi}(s)\rangle. \qquad (1)$$

Thus the geometric phase is essentially the difference between the total phase and the dynamical phase acquired during the evolution.

Consider the evolution of a system consisting of left handed neutrinos ν_L and right handed neutrinos ν_R in presence of magnetic field in matter. Assuming the neutrinos to be propagating along the z-direction, the evolution of a generic neutrino helicity state $\nu = (\nu_L, \nu_R)^T$ can be described by the equation

$$i\frac{\partial}{\partial z}\nu = H\nu, \qquad (2)$$

where $H = H_0 + H_{wk} + H_B$; H_0 is the vacuum Hamiltonian, H_{wk} and H_B are Hamiltonian for neutrino interaction with matter and magnetic field respectively, and we have approximated the distance z along the neutrino direction with time. For neutrino propagation in arbitrary magnetic fields the longitudinal term contributes only to the order $1/\gamma = m_\nu/E$ [4], so we consider only the magnetic field rotating in transverse plane $B_\perp = Be^{i\phi}$. Neglecting the terms proportional to unit matrix, the neutrino evolution equation (2) can be written as [1]

$$i\frac{\partial}{\partial z}\nu = \begin{pmatrix} V(z)/2 & \mu B(z)e^{i\phi(z)} \\ \mu B(z)e^{-i\phi(z)} & -V(z)/2 \end{pmatrix}\nu, \quad (3)$$

where $V = \Delta V - \Delta m^2 A/2E$; $\Delta m^2 = m_R^2 - m_L^2$, A is a function of neutrino mixing angle θ and $\Delta V = V_L - V_R$, V_L and V_R are matter potentials for left and right handed neutrinos respectively. Transforming to frame rotating with the field using

$$\nu = U\psi = \exp(i\sigma_3\phi/2)\psi, \qquad (4)$$

where $\psi = (\psi_L, \psi_R)^T$, we get the evolution equation in rotating frame

$$i\frac{\partial}{\partial z}\psi = \frac{1}{2}\big[(V+\dot{\phi})\sigma_3 + (2\mu B)\sigma_1\big]\psi, \qquad (5)$$

where $\dot{\phi} = d\phi/dz$. For the case of neutrino propagation in matter with constant density and in magnetic field of constant strength and uniform twist i.e. constant V, B and $\dot{\phi}$, (5) can be integrated analytically and we obtain

$$\begin{aligned}\psi(z) =& \exp\Big(-\frac{i}{2}\big((V+\dot{\phi})\sigma_3 + 2\mu B\sigma_1\big)z\Big)\psi(0)\\ =&\Big[\cos\Big(\frac{\delta E_m z}{2}\Big) - \frac{i}{\delta E_m}\Big((V+\dot{\phi})\sigma_3 + \\ & 2\mu B\sigma_1\Big)\sin\Big(\frac{\delta E_m z}{2}\Big)\Big]\psi(0),\end{aligned} \qquad (6)$$

where $\delta E_m = \sqrt{(V+\dot{\phi})^2 + (2\mu B)^2}$ gives the energy splitting of the eigenstates. Finally, us-

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ing (4) we obtain the evolution equations for the neutrino helicity states

$$\nu_L(z) = e^{i\phi/2}\Big[\big(\cos(\delta E_m z/2) - i\cos 2\theta_m \sin(\delta E_m z/2)\big)\psi_L(0) - i\sin 2\theta_m \sin(\delta E_m z/2)\psi_R(0)\Big] \tag{7}$$

$$\nu_R(z) = e^{-i\phi/2}\Big[\big(\cos(\delta E_m z/2) + i\cos 2\theta_m \sin(\delta E_m z/2)\big)\psi_R(0) - i\sin 2\theta_m \sin(\delta E_m z/2)\psi_L(0)\Big], \tag{8}$$

where θ_m is the mixing angle between ν_L and ν_R given by $\tan 2\theta_m = -2\mu B/(V + \dot{\phi})$. If a beam of left handed neutrinos start at $z = 0$, the transition probability at distance z is given by

$$P(\nu_L \to \nu_R; z) = |\langle \nu_R(z) \,|\, \nu_L(0)\rangle|^2 = \sin^2 2\theta_m \sin\left(\frac{\delta E_m z}{2}\right). \tag{9}$$

Thus the neutrino propagation in magnetic field result in oscillation in the $\nu_L - \nu_R$ basis. The geometric phase associated with the non-adiabatic, non-cyclic evolution of the state $|\nu_L\rangle$ is the given by

$$\begin{aligned}\phi_g^{LL}(z) =& arg\,\langle \nu_L(0)|\nu_L(z)\rangle - \\ &\Im \int_0^z \langle \nu_L(z')|\frac{d}{dz'}|\nu_L(z')\rangle\, dz' \\ =& arg\Big(e^{-i[\phi(0)-\phi(z)]/2}\big(\cos(\delta E_m z/2) \\ &- i\cos 2\theta_m \sin(\delta E_m z/2)\big)\Big) \\ &- \Im \int_0^z \left(\frac{i}{2}\frac{d\phi}{dz'} - \frac{i\delta E_m}{2}\cos 2\theta_m\right) dz' \\ =& \frac{1}{2}\delta E_m z \cos 2\theta_m - \\ &\tan^{-1}\left(\cos 2\theta_m tan \frac{\delta E_m z}{2}\right).\end{aligned} \tag{10}$$

Table (I) gives non-cyclic phases for different neutrino states.

Thus as the neutrinos propagate in uniformly twisting magnetic fields, they acquire a non-zero geometric phase. It can be seen that $\phi_g^{LL} = -\phi_g^{RR}$. Thus the resonant spin-flavor conversion $\nu_L \leftrightarrow \nu_R$ results in sign change of the geometric phase which can be used as tool to look for new points of resonant conversion. These points of resonant spin flavor conversion play a lead role in deciding the relative flux of left and right handed neutrinos as the neutrinos propagate in magnetic fields and finally come out of it.

TABLE I: Non-cyclic phases for different neutrino states.

ϕ_g^{RR}	$-\frac{1}{2}\delta E_m z \cos 2\theta_m + \tan^{-1}\left(\cos 2\theta_m tan \frac{\delta E_m z}{2}\right)$.
ϕ_g^{LR}	$-\frac{\pi}{2} - \phi(z) + \frac{1}{2}\delta E_m z \cos 2\theta_m$
ϕ_g^{RL}	$-\frac{\pi}{2} + \phi(z) - \frac{1}{2}\delta E_m z \cos 2\theta_m$

References

[1] C.Giunti, A.Studenikin, Rev.Mod.Phys. 87, 531(2015).
[2] S.Joshi, S.R.Jain, Phys.Lett. B754, 135(2016).
[3] N.Mukunda, R.Simon, Annals Phys. 228, 205(1993).
[4] A.M.Egorov, A.E.Lobanov, A.I.Studenikin, Phys.Lett. B491, 137(2000).