Emergence of new correlations between neutron star radii and nuclear matter properties

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The bulk properties of neutron stars(NSs) are predominantly governed by the equation of state (EoS) of nuclear matter. Nuclear incompressibility coefficient $K(\rho)$, its derivative $M(\rho)$, symmetry energy coefficient $S(\rho)$ and slope of symmetry energy $L(\rho)$ are the important parameters which determine the nuclear matter EoS. These EoS parameters at saturation density(ρ_0) are defined as, $K_0 = 9\rho_0^2 \left(\frac{\partial^2 e(\rho,0)}{\partial \rho^2}\right)_{\rho_0}$, $M_0 = 3\rho_0 \left(\frac{dK}{d\rho}\right)_{\rho_0}$, $J_0 = S(\rho_0)$, where $S(\rho) = \frac{1}{2}\left(\frac{\partial^2 e(\rho,\delta)}{\partial \delta^2}\right)_{\delta=0}$, and $L_0 = 3\rho_0 \left(\frac{\partial S}{\partial \rho}\right)_{\rho_0}$, here $e(\rho,\delta)$ denotes the energy per nucleon at baryon number density ρ and isospin asymmetry $\delta = \frac{\rho_n - \rho_p}{\rho}$, with ρ_n and ρ_p being neutron and proton number density, respectively. Recently, the correlation between the various key parameters of the EoS and the different NS properties have been studied extensively. The correlations of NS radii with $L(\rho)$ are found to be model dependent[1]. In fact, results for different investigations are at variance. In this work, we have studied the correlation of NS radii with the key EoS parameters K_0, M_0, J_0 and L_0 and also with some selected linear combinations of these EoS parameters.

We have taken 22 of relativistic mean field (RMF) models for this correlation study. Two different types of RMF models are used , (1) type I models with nonlinear self and/or mixed interaction terms and constant coupling strengths (BSR2, BSR3, BSR6, BSR10, BSR17, FSU, FSU2, GM1, IUFSU, NL3, NL3$\sigma\rho$4, NL3$\sigma\rho$6, NL3$\omega\rho$02, NL3$\omega\rho$03, TM1, TM1-2) and (2) type II models with only linear interaction terms but density dependent coupling strengths (DD2, DDHδ, DDHδMod, DDME1, DDME2, TW).

The EOSs for β-equilibrated matter over a wide range of density, required to calculate the NS properties, are obtained as follows. The EoS for the outer crust region is taken from the work of Baym-Pethick-Sutherland. For the inner crust region, the EoSs for the pasta phase upto the core-crust transition density ρ_t, are obtained within a Thomas Fermi calculation. The EoSs for the homogeneous core part for $\rho > \rho_t$ are obtained using RMF models. The NS radii are calculated by integrating Tolman-Oppenheimer-Volkoff equations. Once the EoS parameters and NS radii are obtained for all these models, one can find the strength of correlation between these quantities by calculating the Pearson's correlation coefficient $C(a,b)$ given by,

$$C(a,b) = \frac{\sigma_{ab}}{\sqrt{\sigma_{aa}\sigma_{bb}}} \quad (1)$$

with the covariance, σ_{ab}, written as

$$\sigma_{ab} = \frac{1}{N_m}\sum_i a_i b_i - \left(\frac{1}{N_m}\sum_i a_i\right)\left(\frac{1}{N_m}\sum_i b_i\right), \quad (2)$$

TABLE I: The correlation coefficients of NS radius with K_0, M_0, J_0 and L_0 for different NS masses. Subscript of R denotes the mass of NS in $M_\odot$.

	K_0	M_0	J_0	L_0
$R_{0.6}$	0.442	0.370	0.915	0.932
$R_{0.8}$	0.523	0.444	0.876	0.933
$R_{1.0}$	0.601	0.537	0.826	0.906
$R_{1.2}$	0.666	0.638	0.765	0.848
$R_{1.4}$	0.717	0.737	0.681	0.760
$R_{1.6}$	0.740	0.815	0.577	0.641

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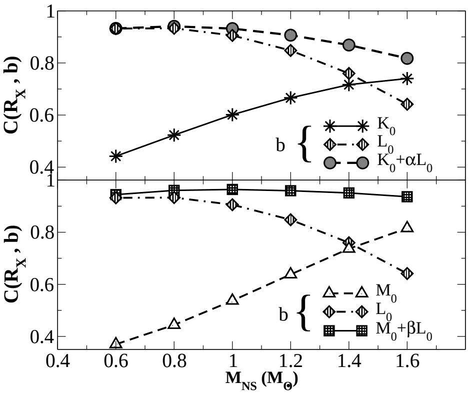

FIG. 1: The correlation coefficients of NS radii with K_0, L_0 and $K_0 + \alpha L_0$ (upper panel), and M_0, L_0 and $M_0 + \beta L_0$ (lower panel) as a function of the NS mass.

where, a_i and b_i correspond to the neutron star radius for a fixed mass and a EoS parameter, respectively, obtained for different models and the index i runs over the number of models $N_m = 22$.

In Table I, the correlation coefficients between these EoS parameters and the radii of NS with different masses are listed. The correlation of NS radii with K_0 and M_0 increase with the increase of mass, but with J_0 and L_0 the trend is opposite. These results indicate that the radius of low mass NS is more sensitive to J_0 and L_0, but, as the NS mass increases, the sensitivity to K_0 and M_0 tend to dominate. Thus, no single EoS parameter shows strong correlation with NS radii over a wide range of NS mass.

We also look into the correlations of NS radii with the linear combinations of EoS parameters $K_0 + \alpha L_0$ and $M_0 + \beta L_0$. The values of α (β) are obtained in such a way that the correlations of the neutron star radii with $K_0 + \alpha L_0$ ($M_0 + \beta L_0$) are the maximum. The upper panel of Fig.1 shows the variation of the correlation coefficients of NS radii with K_0, L_0 and $K_0 + \alpha L_0$ as a function of NS mass(M_{NS}) . The lower panel of the figure shows the same, but for M_0, L_0 and $M_0 + \beta L_0$. The correlation of NS radii with $K_0 + \alpha L_0$ is better than those with K_0 and L_0 individually. However, for $M_{NS} \gtrsim 1.0 M_\odot$, the correlations of NS radii decreases gradually with the increase of the NS mass, even considering $K_0 + \alpha L_0$. On the other hand, the NS radii are strongly correlated with $M_0 + \beta L_0$ over a wide range of NS mass (0.6-1.6$M_\odot$). For $M_{NS} \sim 1.4 M_\odot$, significant improvement in the correlations of NS radii with these linear combinations of EoS parameters is observed compared to those with the individual parameters. We trace back to the underlying reason for the strong correlation between NS radii and $M_0 + \beta L_0$. Emergence of such strong correlation is due to the fact that the pressure is strongly correlated with the linear combination of M_0 and L_0 in the density range ρ_0–$2\rho_0$[2]. In Table II, we have listed the correlation coefficient of NS radii with $K_0 + \alpha L_0$ and $M_0 + \beta L_0$ along with the values of α and β, for different neutron star masses.

TABLE II: The correlation coefficients of neutron star radii with $K_0 + \alpha L_0$ and $M_0 + \beta L_0$, along with the values of α and β.

	$K_0 + \alpha L_0$		$M_0 + \beta L_0$	
	α	Corr. Coeff.	β	Corr. Coeff.
$R_{0.6}$	27.35	0.933	119.71	0.944
$R_{0.8}$	6.27	0.941	78.05	0.961
$R_{1.0}$	3.09	0.932	50.64	0.964
$R_{1.2}$	1.82	0.907	33.40	0.959
$R_{1.4}$	1.11	0.869	22.00	0.951
$R_{1.6}$	0.65	0.818	14.12	0.936

To conclude, the EoS parameters individually show good correlation with the NS radii only in the narrow range of mass. For instance, EoS parameters J_0 and L_0 are well correlated with the radii of the low mass NS and the EoS parameters K_0 and M_0 show good correlation with radii of NS with higher mass. However, our investigation suggests the emergence of a very strong correlation of NS radii with the linear combination of the slope of nuclear incompressibility M_0 and the slope symmetry energy L_0, and such correlation is practically independent of NS mass over a wide range (0.6-1.6$M_\odot$).

References

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