Curvature of a neutron star

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Introduction

In the universe, neutron stars(NSs) are excellent laboratories for determining the equation of state (EOS) of cold dense matter. We can not create such a high density in terrestrial laboratory, so a neutron star is and the only object, which can provide much information on high-density nature of the matter. But it is not an easy task to deal with the neutron star for it's complex nature, as all the four fundamental forces (strong, weak, gravitational and electromagnetic) are active. For high gravitational field, the general theory of relativity is used, where exist a set of invariant curvature, as they are scalars. They can be formed from the Riemann, Ricci, and Weyl tensors. These invariant curvatures describe the physical properties of the space-time. Using these definition, we calculate the curvature[1] of the neutron star.

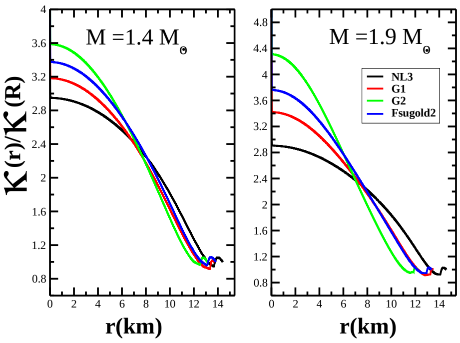

FIG. 2: The ratio of normalized curvature profile $\mathcal{K}$(r) and the surface curvature $\mathcal{K}$(R) of the neutron stars for various EOSs.

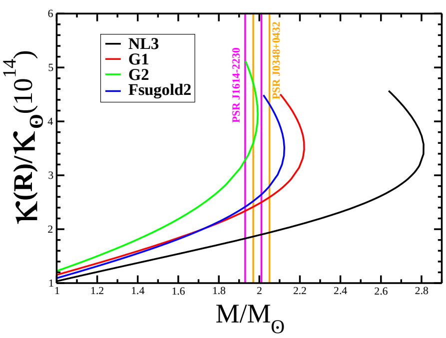

FIG. 1: The ratio of surface curvatures of neutron stars and the Sun as a function of the NSs masses for different EOSs.

Results and Discussions

We employ the EOSs[2] to calculate the mass and radius profile of the static (non-rotating) neutron star. We solve the Tolmann-Oppenheimer-Volkov (TOV) equations which are written as:

$$\frac{dP(r)}{dr} = -\frac{[\mathcal{E}(r) + P(r)][M(r) + 4\pi r^3 P(r)]}{r^2(1 - \frac{2M(r)}{r})} \quad (1)$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \mathcal{E}(r), \quad (2)$$

using G=c=1. Where, $\mathcal{E}(r), P(r)$ and $M(r)$ are the energy density, pressure and mass profile respectively as a function of radial distance r. These differential equations solved for the given boundary conditions $\mathcal{E}(0) = \mathcal{E}_c$, $M(0) = 0, P(R) = 0$, and $M(R) = M$. Next,

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$$\mathcal{K}^2 = \mathcal{W}^2 + 2\mathcal{J}^2 - \frac{1}{3}\mathcal{R}^2 \quad (3)$$

where, $\mathcal{W}^2$ is the full contraction of the Weyl tensor, defined as

$$\mathcal{W}^2 \equiv \mathcal{C}^{\mu\nu\rho\sigma}\mathcal{C}_{\mu\nu\rho\sigma} = \frac{4}{3}\big(\frac{6M(r)}{r^3} - \kappa\mathcal{E}(r)\big)^2. \quad (4)$$

The full contraction of the Ricci tensor is

$$\mathcal{J}^2 \equiv= \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} = \kappa^2\big[\mathcal{E}^2(r) + 3P^2(r)\big] \quad (5)$$

and the Ricci scalar is

$$\mathcal{R}(r) = \kappa\big(\mathcal{E}(r) - 3P(r)\big), \quad (6)$$

with $\kappa = 8\pi$. We noticed that the components of the Ricci tensor $\mathcal{R}_{\mu\nu}$ and the Ricci scalar $\mathcal{R}$ vanish outside the star. Because at the surface of the star, the pressure P and energy density $\mathcal{E}$ vanishes. The only nonvanishing component of the Riemann tensor contributes at the surface. Thus the full contraction of the Riemann tensor is a suitable measure of the curvature for the space-time than the Ricci scalar and the Ricci tensor[1].

In Fig.1, we have plotted the ratio of the surface curvatures of neutron star with the Sun ($\mathcal{K}_\odot \equiv 3.0$ x 10^{-17} km^{-2}) as a function of the NSs masses for different EOSs. Mass of the NS in the case of G2 parameter is found in the region of current observations. The surface curvature of the solar system is smaller by 14 order[3] of magnitude then the NSs curvature. The NSs surface curvature $\mathcal{K}_R \approx$ 3-4 x $10^{14}\mathcal{K}_\odot$ has been obtained for massive NSs in the range of 1.97< M/M$_\odot$ <2.8. The small variation in $\mathcal{K}_R$ ~1-2 x $10^{14}\mathcal{K}_\odot$ of canonical neutron stars predicted by the EOSs. Notice that for larger massive star (as predicted by NL3 type interaction) has less surface curvature as compared to the smaller massive star given by G2 type force parameter. This is more clear in the Fig. 2, which gives the complete profile of the normalized curvature of the neutron stars of masses between 1.4 M$_\odot$ and 1.9 M$_\odot$. The normalized curvature $\mathcal{K}(r) \approx$3.0-3.6 $\mathcal{K}(R)$ is noticed to be larger near the centre of the star (See left panel of Fig. 2) and suddenly decreases when it goes away from the centre. This means that the curvature of the neutron star in the core is larger, because of the existence of quark like particles. The normalized curvature is lower in the case of 1.9 M$_\odot$ with an exception with the prediction of NL3 set. This effect is due to the higher compressibility and symmetry energy of this force parameter.

Summary and Conclusion

We have calculated the curvature of NSs using the square root of the full contraction of the Riemann tensor. We found the NSs surface curvature $\mathcal{K}(R)$ is very large of the order 10^{14} comparison to the Solar system and also the complete picture of the normalized curvature found to be very large near the center of the star. We noticed a significant effect of compressibility and symmetry energy on the curvature of neutron star.

References

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