Astrophysical S-factor for sub-barrier Fusion Reaction

Vinay Singh[1],* Debasis Atta[1,2], and Md A Khan[3]
[1] *Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700064, INDIA*
[2] *Govt. General Degree College, Kharagpur II, West Bengal 721 149, INDIA and*
[3] *Dept. of Physics, Aliah University, IIA/27, New Town, Kolkata-700156, INDIA*

Introduction

The nuclear fusion at very low energies (∼1eV to few keV) plays important role in nucleosynthesis of light elements in stellar core. Nuclear fusion reaction in this energy range can be explained successfully by the phenomenon of quantum mechanical tunneling through Coulomb barrier of interacting nuclide. In the present work, a simple square-well potential model with an imaginary part is used to describe the d+t nuclear fusion reaction where the real part of the potential is mainly derived from the resonance energy while the imaginary part is determined by the Gamow factor at resonance energy.

Theoretical framework

When a deuteron is injected to a triton, the relative motion can be described in terms of the reduced radial wave function $\zeta(r)$ given by $\Phi(r,t) = \frac{1}{\sqrt{4\pi r}}\zeta(r)\exp(-i\frac{E}{\hbar}t)$ where $\Phi(r,t)$ represents the solution of the general Schrödinger equation for the system. The reaction cross section in terms of the phase shift δ_0 due to the nuclear potential (in low energy limit only s-wave contributes) is given by $\sigma = \frac{\pi}{k^2}[1-|\eta|^2]$ where $\eta = e^{2i\delta_0}$ and k is the wave number for relative motion. Nuclear potential being complex, the corresponding phase shift δ_0 is complex and given by [1]

$$cot(\delta_0) = W_r + iW_i. \tag{1}$$

Consequently, the cross section is given by

$$\begin{aligned}\sigma &= \frac{\pi}{k^2}\left\{-\frac{4W_i}{(1-W_i)^2+W_r^2}\right\} \quad (2)\\ &= \left(\frac{\pi}{k^2}\right)\left(\frac{1}{\chi^2}\right)\left\{-\frac{4\omega_i}{\omega_r^2+(\omega_i-\frac{1}{\chi^2})^2}\right\}\end{aligned}$$

where $\chi^2 = \left\{\frac{\exp(\frac{2\pi}{ka_c})-1}{2\pi}\right\}$ is the Gamow penetration factor. The quantity $S(E)$ given by

$$S(E) = \left\{-\frac{4\omega_i}{\omega_r^2+(\omega_i-\frac{1}{\chi^2})^2}\right\} \tag{3}$$

is called the astrophysical S-factor, where $\omega = \omega_r + i\omega_i = W/\chi^2 = (W_r+iW_i)/\chi^2$. The wave function inside the nuclear well is determined by two parameters, the real V_r and the imaginary part V_i of the nuclear potential. The Coulomb wave function outside the nuclear well is determined by two other parameters: the real and the imaginary part of the complex phase shift $(\delta_0)_r$ and $(\delta_0)_i$. A pair of convenient parameters, W_r and W_i, are introduced to make a linkage between the cross section and the nuclear well so that it is easy to discuss the resonance and the selectivity in damping. The boundary condition for the wave function can be expressed by its logarithmic derivative which for the square well is given by

$$R\frac{[\sin(Kr)]'}{\sin(Kr)}|_{r=R} = KR\cot(KR) \tag{4}$$

and in the Coulomb field, it is given by

$$\frac{R}{a_c}\left\{\frac{1}{\chi^2}\cot(\delta_0) + 2\left[\ln\left(\frac{2R}{a_c}\right)+2A+y(ka_c)\right]\right\} \tag{5}$$

so that

$$\begin{aligned}\omega_i &= W_i/\chi^2 = Im\left[\frac{a_c}{R}(KR)\cot(KR)\right] \quad (6)\\ &= \frac{a_c}{R}\left\{\frac{\gamma_i\sin(2\gamma_r)-\gamma_r\sinh(2\gamma_i)}{2[\sin^2(\gamma_r)+\sinh^2(\gamma_i)]}\right\}\\ \omega_r &= W_r/\chi^2 = \frac{a_c}{R}\left\{\frac{\gamma_r\sin(2\gamma_r)+\gamma_i\sinh(2\gamma_i)}{2[\sin^2(\gamma_r)+\sinh^2(\gamma_i)]}\right\}\\ &\quad - 2H \quad (7)\end{aligned}$$

*Electronic address: vsingh@vecc.gov.in

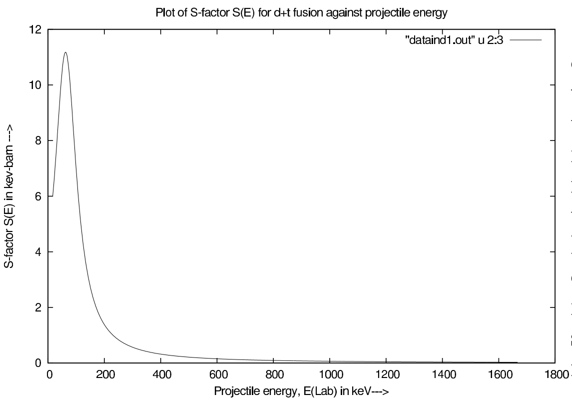

FIG. 1: S-factor S(E) in keV-barn for d+t fusion reaction using Eq.(3).

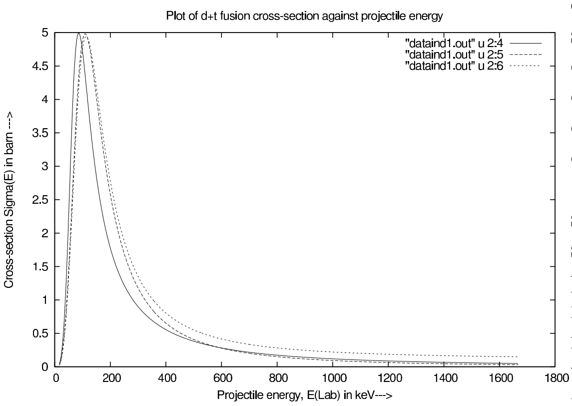

FIG. 2: Comparison of fusion cross sections obtained using Eq.(2) and those obtained using three and five parameter fitting formulas of Ref.[2].

where $K^2=\frac{2\mu}{\hbar^2}[E-(V_r+iV_i)]$, $K_i=\frac{\mu}{K_r\hbar^2}(-V_i)$, $\gamma=(\gamma_r + i\gamma_i) \equiv (K_rR + iK_iR)$, $k^2=\frac{2\mu E}{\hbar^2}$, $H=\left[\ln\left(\frac{2R}{a_c}\right)+2A+y(ka_c)\right]$, $a_c=\frac{\hbar^2}{Z_1Z_2\mu e^2}$ and $y(x)=\frac{1}{x^2}\sum_{j=1}^{\infty}\frac{1}{j(j^2+x^{-2})}$. In the above relations k is the wave number outside the nuclear well, a_c is the Coulomb unit of length, and A=0.577 is Euler's constant, $y(ka_c)$ is related to the logarithmic derivative of Γ function given as y(x). The cross-section can also be computed using the three and five parameter fitting formulas [2].

Calculations and Results

The fusion cross sections and the astrophysical S-factors are calculated using Eq.(2) and Eq.(3), respectively. The results of present calculations for S-factors are shown in Fig.1 while present calculation results for cross sections are compared with those calculated using the three and five parameter fitting formulas of Ref.[2] in Fig.2. The experimental data and the quantum-mechanical calculations show good agreement. However, calculations of fusion cross sections for reactions involving medium and heavy nucleus-nucleus systems, a completely different approach is needed [3].

Summary and Conclusion

The complex potential causes absorption of the projectile into the nuclear well. For over last six decades controlled nuclear fusion research has been concentrated much on deuteron-triton fusion because of their large cross-section compared to that of deuteron-deuteron fusion by a factor of several hundred in spite of both having almost the same Coulomb barrier. The resonance of the d+t state near 100 keV is considered as the reason for such a large cross section. A simple square-well potential model with an imaginary part can be used to describe the d+t nuclear fusion reaction. It is interesting to notice that while the real part of the potential is mainly derived from the resonance energy, the imaginary part of the potential is determined by the Gamow factor at resonance energy. The good agreement between the experimental data and the quantum-mechanical calculation suggests a selective resonant tunneling model, rather than the conventional compound nucleus model, because the penetrating particle will keep its memory of the phase factor of its wave function. The implication of this selective resonant tunneling model can be further explored for other light nuclei fusion.

References

[1] Xing Zhong Li, Jian Tian, Ming Yuan Mei, Chong Xin Li, Phys. Rev. **C 61**, 024610 (2000).

[2] Xing Z. Li, Qing M. Wei, Bin Liu, Nucl. Fusion **48**, 125003 (2008).

[3] Debasis Atta, D. N. Basu, Phys. Rev. **C 90**, 064622 (2014).