Understanding Muon Scattering Using GEANT4 for Tomographic Application

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Introduction

Cosmic-ray muons are the most numerous charged particles at sea level. Their flux at sea level is around $10000/m^2/min$, that changes with the zenith angle (θ) as $\cos^2\theta$. Their energy varies from 10 MeV to 10 GeV with most probable value around 1 GeV. As charged particles, they interact with matter via ionization of atoms and molecues and Coulomb scattering from nuclei. Muon Tomography is a method of producing three-dimensional (3D) image of the medium utilizing the information of the interaction of muon with the medium.

Muon tomography can be done in two ways; one is muon absorption tomography and the other one is muon scattering tomography (MST). In the earlier case, energy loss of muon is recorded using suitable detectors and the intensity of an image pixel is determined by the attenuation of incident muons caused by their absorption in matter; whereas in the latter case, the amount of deviation of the muons tracks due to Coloumb scattering is determined by placing layers of detectors above and below the medium. The scattering map based on the amount of scattering and its location, provides a basis of image pixel. In case of MST, the reliability of the method depends upon accurate estimation of scattering angles. Several algorithms have been introduced to estimate this. In the present work, a calculation has been performed to analyze the efficacy of one such algorithm, namely POCA (Point Of Closest Approach), by comparing it with a detailed model calculation done with GEANT4.

GEANT4 simulation

Muons undergo multiple deflections due to Coulomb scattering from the atomic neuclei of the medium. The scattering angle distribution can be approximated to a Gaussian distribution [1], with mean at zero and root mean square (RMS) width given by σ_{mcs}.

$$\sigma_{mcs} = \frac{13.6}{\beta cp}\sqrt{\frac{L}{X_0}}(1 + 0.038ln(\frac{L}{X_0})) \qquad (1)$$

where L is the length of the medium traversed, β is the ratio of speed of the muon to speed of light and p is the momentum of the muon. The radiation length (X_0) is given by

$$X_0 = \frac{716.4g/cm^2}{\rho}\frac{A}{Z(Z+1)ln(\frac{287}{\sqrt{Z}})} \qquad (2)$$

It can be clearly seen from above equations that σ_{mcs} varies directly with $\sqrt{\rho}Z$ where Z is the atomic number and ρ is the density of the medium. In GEANT4 simulations, muons of energy 1 GeV were incident normally on cuboid volumes (10 cm×10 cm×10 cm) of several materials, and mean scattering angle (θ_m) and RMS width (σ_{mcs}), hence found are given in Table-1.

Later, variation with size of the material and incident muon energy was studied. As the size increases, the muons scatter more and as the muon energy increases, the scattering probability reduces. Keeping these two parameters constant, the scattering angle distribution with $\sqrt{\rho}Z$ was studied. Figure-1 displays the variation of RMS width (σ_{mcs}) with $\sqrt{\rho}Z$ to be linear for four materials.

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Materials(Z)	$\rho(g/cc)$	θ_m(mrad)	σ_{mcs}(mrad)
U(92)	18.95	111.2	60.1
Pb(82)	11.35	79.8	42.7
Fe(26)	7.87	42.4	23.8
Al(13)	2.702	17.5	10.7
Concrete (Z/A = 0.5)	2.3	19.5	12.8

TABLE I: The mean scattering angle, RMS width simulated with GEANT4 for several materials

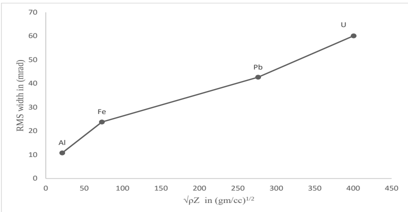

FIG. 1: The dependence of scattering angle distribution with $\sqrt{\rho}Z$ for Aluminum, Iron, lead and Uranium.

Comparison of GEANT4 and POCA

In GEANT4, a model was created keeping two Bakelite RPC detectors; one above and the other below the target medium. The interaction points with gas volume were taken as the detector hits without taking the position resolution into account, and the hits were used to build incoming and outgoing tracks. On the other hand, a simple geometric algorithm namely, Point Of Closest Approach (POCA) was considered. It ignores multiple coulomb scattering and assumes a muon to get scattered at a single point. The mid-point of the line joining the closest points of approach of the projected incoming and outgoing tracks is calculated. Which is considered as the scattering point where the deviation between the incoming and outgoing tracks is determined as scattering angle. In the current work, the POCA calculated points are compared to the actual points obtained from GEANT4 simulation and the distance between them is estimated. The residual plot of POCA with respect to GEANT4 is shown in figure-2. For a lead block of dimensions (20 cm×20 cm×20 cm) it shows that residual of GEANT4 with respect to POCA is around 2.583 cm.

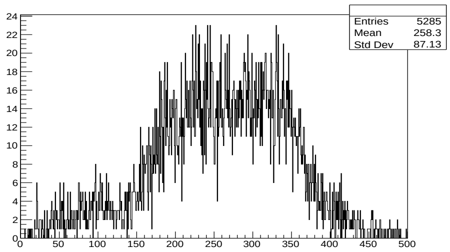

FIG. 2: Residual plot of POCA with respect to GEANT4

Summary and Future plans

The comparison of the scattering points produced with a GEANT4 model based on realistic physical processes and POCA based on geometrical approximations shows that their residual is fairly large. It is planned to include Cosmic Ray Library (CRY) developed by LANL [2] which generates correlated cosmic muons taking care of their energy, incoming time and zenith angle dependence, as the library into the GEANT4 code. An experimental setup is also under construction to perform the measurements of scattering angles.

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References

[1] W. T. Scott, *Mean-Value Calculations for Projected Multiple Scattering*, Rev. Mod. Phys. 35 (1963) 231

[2] J A Green et al., 2006 IEEE Nuclear Science Symposium Conference Record.