Statistical ensembles and fragmentation of finite nuclei

G. Chaudhuri,* P. Das, and S. Mallik
[1] *Theoretical Nuclear Physics Group, Variable Energy Cyclotron Centre, HBNI, 1/AF Bidhan Nagar, Kolkata 700 064, INDIA*

The statistical models based on different statistical ensembles (micro canonical, canonical and grand canonical) have been widely used to describe the nuclear multifragmentation reaction in heavy ion collisions ate intermediate energy [1, 2]. The basic assumption behind this is attainment of statistical equilibrium at the freeze out stage. In such models of nuclear disassembly, the populations of different channels is solely decided by their statistical weights in available phase space. The micro canonical ensemble is applicable in the case of fixed particle number and fixed energy but any practical calculation based on this is extremely difficult because of these two constraints. The canonical model on the other hand is applicable when the number of particles is finite (as would be in experiments) but the energy is varying though the average number is constrained to a given value [2]. The grand canonical on the other hand is for both varying particle number as well as energy. The grand canonical version of the model [3] for nuclear multifragmentation has been known for long time and is the most commonly used. But it is more important to know how to treat an exact number of particles rather than an ensemble of particle numbers since the given dissociating system (finite nucleus) has a fixed number of particles. The answer is the canonical ensemble which deals with a given number of baryon and lepton numbers. But these constraints of baryon and lepton number conservation put severe restrictions on calculation of the partition sum. This led to the more frequent use of the grand canonical ensemble for describing the fragmentation of finite nuclei. The main motivation of this work is to formulate a transformation relation so that results from one ensemble can be converted to the other easily with the help of such relation. Consider the values of any observable in canonical and grand canonical ensembles are $R_c(N_0, Z_0)$ and $R_{gc}(f_n, f_p)$ respectively at a given temperature and freeze-out volume. N_0 and Z_0 are total number of neutrons and protons for canonical case and for grand canonical case these are average numbers at fugacities f_n and f_p respectively(total number can fluctuate as N and Z). Then the canonical value of the observable can be expressed can be expressed in terms of grand canonical model outputs only from the relation

$$R_c(N_0, Z_0) \approx R_{gc}(f_n, f_p) + \frac{1}{2}\sigma_{f_n}^2 \frac{\partial^2 R_{gc}}{\partial N^2}\bigg|_{N_0,Z_0} + \frac{1}{2}\sigma_{f_p}^2 \frac{\partial^2 R_{gc}}{\partial Z^2}\bigg|_{N_0,Z_0} + \sigma_{f_n f_p} \frac{\partial^2 R_{gc}}{\partial N \partial Z}\bigg|_{N_0,Z_0} \quad (1)$$

where σ's are the particle variances. With this one can avoid the computer intensive partition sum calculation of the canonical ensemble and directly arrive at the results from the grand canonical ensemble ones.

We have also examined the conditions of convergence of canonical and grand canonical ensembles under different conditions. It is well known that results from canonical and grand canonical ensembles agree in the thermodynamic limit that is when the number of particles become infinite. But in the case of finite nuclei too, they converge under certain conditions [4]. These conditions of equivalence or convergence can be easily tested using the above transformation relation [Eq. (1)]. It is observed that results from both the ensembles converge more and more if either temperature or source size or freeze out volume is increased or asymmetry of the fragmenting nucleus is decreased. The

*Electronic address: gargi@vecc.gov.in

importance of the transformation formula is that it can be used to extract results from canonical model using those from grand canonical in the domain where the results from these two ensembles do not converge.

Different fragmentation observables have been examined in order to test the predictability of the formula developed for converting results from grand canonical to canonical ones. The isotopic distribution of Z =7 nucleus is shown in the adjacent figure and three results are shown, one is from the canonical model, another is that from grand canonical model and the third is the results from the transformation formula converting results from grand canonical to canonical. It is seen that results from the transformation formula agree with that of the canonical to a large extent The other observables which have also been tested are mass distribution at different temperatures, the size of the largest cluster and in each case conclusion is similar.

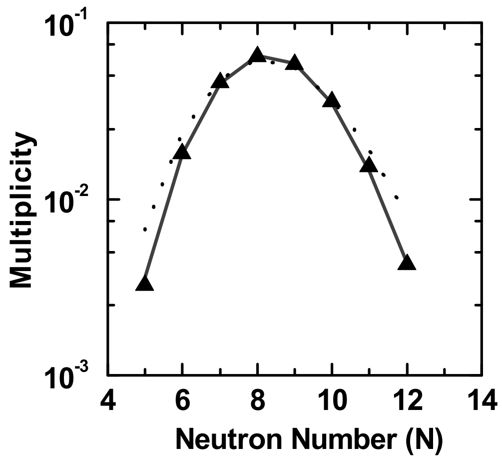

FIG. 1: Multiplicities of Z =7 isotopes produced from the fragmenting system Z_0 =28, A_0 =64 calculated from canonical (solid line) and grand canonical (dotted line) model. The triangles represent the canonical result obtained from grand canonical model by using Eq. 1.

Another important domain where this transformation relation can be immensely useful is while dealing with isoscaling [5] and isobaric yield ratio parameters [6] and also temperature measurement by double isotope ratio method [7]. These isoscaling and isobaric yield ratio equations as well as the equation for measuring the temperature have been derived using the yields of the fragments in the framework of grand canonical ensemble. Hence their applicability in case of finite nuclei is limited and is not valid for all energies, source size as well as asymmetry ratio [8]. These aspects can be tested since through the transformation relations one can also derive easily the results in the corresponding canonical ensemble. Also using the inverse set of transformation relations one can easily switch over to grand canonical results from a given set of canonical ones.

The transformation relations connecting the two ensembles is not valid in the temperature or density regime where the liquid gas phase transition occurs. Fluctuations become extremely high in this domain which limits the applicability of the formula. One should not also apply the formula in the region when cross section is very small since in those region higher order corection terms to the formula should be taken into account.

References

[1] J. P. Bondorf et. al., Phys. Rep. **257**, 133 (1995).

[2] C. B. Das, S. Das Gupta et al., Phys. Rep. **406**, 1 (2005).

[3] G. Chaudhuri and S. Das Gupta, Phys. Rev. C **80** 044609 (2009).

[4] S. Mallik and G. Chaudhuri, Phys. Lett. B **718**, 189 (2012).

[5] M. B. Tsang et al., Phys. Rev. Lett. **86**, 5023 (2001).

[6] M. Huang et. al., Phys. Rev. C **81** 044620 (2010).

[7] S. Albergo et. al., Nuovo Cimento A **89**, 1 (1985).

[8] S. Mallik and G. Chaudhuri, Phys. Let. B **727**, 282 (2013).