Role of asymmetry in breakdown in $N_pN_n$ scheme

Yuvraj Singh$^1$, M. Singh$^2$, A. K. Vrshney$^3$, K. K. Gupta$^4$

$^1$Govt. College, Shahpur (HP), INDIA
$^2$Greater Noida Institute of Engineering, Greater Noida – 201308 (UP), INDIA
$^3$Govt. College, Palampur (HP), INDIA
$^4$Govt. College, Dhaliara (HP), INDIA

*Email: ypchingi@gmail.com

The rigid triaxial rotor model (RTRM) considers the nucleus as a rigid rotor with rigid triaxial asymmetry $\gamma$. For a fixed value of deformation parameter ($\beta$) violation of axial symmetry of the nucleus leads to an increase of energy of the levels belonging to the axial nucleus in the Davydov Filippov model [1]. The increase of level energy is corresponds to the decrease of effective moment of inertia of the nucleus. For the first excited state of spin 2 the effective moment of inertia is determined from the relation –

$$E_{2\gamma}^+ = \frac{\hbar^2}{2I_{0}} \left[ a(-1)\sigma_{12}/[1-\gamma^{2}\sin^{2}3\gamma] \right]$$

Where $\sigma_{12} = 0, 1$. The reduced E2 transition rate from the $2^+_{\gamma}$ states to the ground state can be expressed as –

$$B(E2; 2_{\gamma}^+ \rightarrow 0^+_1) = \frac{1}{2} \left( \frac{\gamma^2 \delta_{0}^2}{16 \pi} \right) \left[ 1 + \left( -1 \right)^{\sigma_{12}} \frac{2-2\sigma_{12}^{2} \sin^{2}3\gamma}{\sqrt{9-8\sin^{2}3\gamma}} \right]$$

Where $Q_0 = \frac{3\pi \gamma^2 \delta \beta}{\sqrt{8\pi}}$

And the value of $B(E2; 2_2^+ \rightarrow 0^+_1)$ is given by –

$$B(E2; 2_2^+ \rightarrow 0^+_1) = \frac{10}{7} \left( \frac{\gamma^2 \delta_{0}^2}{16 \pi} \right) \left[ \frac{\sin^{2}3\gamma}{9-8\sin^{2}3\gamma} \right]$$

In the present work, we evaluate the values of $\gamma$ for even – even Hf nuclei from equations 1, 2 and 3. The asymmetry parameter $\gamma$ are calculated from the energy ratio $\frac{E_{2\gamma}^+}{E_{22}^+}$ are written as $\gamma_e$ while calculated from B (E2) branching ratio $\frac{B(E2; 2_{\gamma}^+ \rightarrow 2^+_1)}{B(E2; 2_{2}^+ \rightarrow 0^+_1)}$ are written as $\gamma_b$.

We keep in mind that although the Hf nuclei are known to be $\gamma$ – soft and RTRM embodies a nuclear shape with rigid triaxiality, the expectation or rms values of $\gamma$ should be valid. In the $N_pN_n$ scheme the interactive forces inside the nucleus are said to be proportional to the product $N_pN_n$. The product is proportional to the B (E2) transition value $B(E2; 2_1^+ \rightarrow 0^+_1)$ and to the level energy $E_{21}^+$. In table – I, we observe that in $^{164-170}$Hf nuclei the values of $N_pN_n$ increase so the B (E2) values, while $E_{21}^+$ values decrease. Thus $N_pN_n$ scheme is followed. For $^{164-168}$Hf nuclei, the values of $\gamma$ calculated in different ways from energy ratios ($\gamma_e$) and E2 transition ratio ($\gamma_b$) are almost equal, but in $^{170}$Hf the $\gamma$ values are quite different ($\gamma_e = 12.8, \gamma_b = 25.7$). Therefore, the internal consistency of RTRM is found to be disturbed. In $^{176}$Hf nucleus a sudden breakdown in $N_pN_n$ scheme appears.

The B (E2) value decreases instead of increasing with the increase of $N_pN_n$. At the same time the value of $\gamma_b$ is also reduced to 18.0 from 25.6 just, lowering the gap between the $\gamma_e$ and $\gamma_b$. In the next nucleus $^{174}$Hf the difference between $\gamma_e$ and $\gamma_b$ vanishes and B (E2) starts increasing again with the increase in $N_pN_n$. It continues further in $^{176}$Hf nucleus.

In the above observations it is clear that the erratic value in $\gamma$ has some role in starting the breakdown in $N_pN_n$ scheme. The erratic value of $\gamma$ challenges the internal consistency of rigid triaxial rotor model and also bring breakdown in $N_pN_n$ scheme. The above systematic repeated again in $N_pN_n$ scheme where the difference between $\gamma_e$ and $\gamma_b$ is large in $^{180}$Hf and the breakdown is followed from $^{182}$Hf.
The \( B(E2) \) value for \( ^{182}\text{Hf} \) is evaluated employing Grodzins [4] relation
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\frac{\Delta E2^1}{\Delta E2^2} = (2.5 + 1) \times 10^{-3} \ MeV \cdot e^2b^2
\]

References:

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