

## Role of asymmetry in breakdown in $N_p N_n$ scheme

Yuvraj Singh<sup>1\*</sup>, M. Singh<sup>2</sup>, A. K. Vrushney<sup>3</sup>, K. K. Gupta<sup>4</sup>

<sup>1</sup>Govt. College, Shahpur (HP), INDIA

<sup>2</sup>Greater Noida Institute of Engineering, Greater Noida – 201308 (UP), INDIA

<sup>3</sup>Govt. College, Palampur (HP), INDIA

<sup>4</sup>Govt. College, Dhaliara (HP), INDIA

\*Email: [vpchingi@gmail.com](mailto:vpchingi@gmail.com)

The rigid triaxial rotor model (RTRM) considers the nucleus as a rigid rotor with rigid triaxial asymmetry  $\gamma$ . For a fixed value of deformation parameter ( $\beta$ ) violation of axial symmetry of the nucleus leads to an increase of energy of the levels belonging to the axial nucleus in the Davydov Filippov model [1]. The increase of level energy is corresponds to the decrease of effective moment of inertia of the nucleus. For the first excited state of spin 2 the effective moment of inertia is determined from the relation –

$$E 2_{1,2}^+ = \left(\frac{6\hbar^2}{2\theta_0}\right) \frac{a-(-1)2\sqrt{81-72\sin^2 3\gamma}}{4\sin^2 3\gamma} \quad (1)$$

Where  $\sigma_{1,2} = 0, 1$ . The reduced E2 transition rate from the  $2_{1,2}^+$  states to the ground state can be expressed as –

$$B(E2; 2_{1,2} \rightarrow 0_1^+) = \frac{1}{2} \left(\frac{e^2 Q_0^2}{16\pi}\right) \left[1 + (-1)^{\sigma_{1,2}} \frac{3-2\sin^2 3\gamma}{\sqrt{9-8\sin^2 3\gamma}}\right] \quad (2)$$

$$\text{Where } Q_0 = \frac{3ZR^2\beta}{\sqrt{5\pi}}$$

And the value of  $B(E2; 2_2 \rightarrow 0_1^+)$  is given by –

$$B(E2; 2_2 \rightarrow 0_1^+) = \frac{10}{7} \left(\frac{e^2 Q_0^2}{16\pi}\right) \left[\frac{\sin^2 3\gamma}{9-8\sin^2 3\gamma}\right] \quad (3)$$

In the present work, we evaluate the value of  $\gamma$  of even – even Hf nuclei from equations 1, 2 and 3. The asymmetry parameter  $\gamma$  are calculated from the energy ratio  $\left(\frac{E 2_2^+}{E 2_1^+}\right)$  are written as  $\gamma_e$  while calculated from B (E2) branching ratio  $\left[\frac{B(E2; 2_2 \rightarrow 2_1^+)}{B(E2; 2_2 \rightarrow 0_1^+)}\right]$  are written as  $\gamma_b$ . We keep in mind that although the Hf nuclei are known to be  $\gamma$  – soft and RTRM embodies a

nuclear shape with rigid triaxiality, the expectation or rms values of  $\gamma$  should be valid. In the  $N_p N_n$  scheme the interactive forces inside the nucleus are said to be proportional to the product  $N_p N_n$ . The product is proportional to the B (E2) transition value  $B(E2; 2_1 \rightarrow 0_1^+)$  and to the level energy  $E 2_1^+$ .

In table – I, we observe that in <sup>164-170</sup>Hf nuclei the values of  $N_p N_n$  increase so the B (E2) values, while  $E 2_1^+$  values decrease. Thus  $N_p N_n$  scheme is followed. For <sup>164-168</sup>Hf nuclei, the values of  $\gamma$  calculated in different ways from energy ratios ( $\gamma_e$ ) and E2 transition ratio ( $\gamma_b$ ) are almost equal, but in <sup>170</sup>Hf the  $\gamma$  values are quite different ( $\gamma_e = 12.8, \gamma_b = 25.7$ ). Therefore, the internal consistency of RTRM is found to be disturbed. In <sup>172</sup>Hf nucleus a sudden breakdown in  $N_p N_n$  scheme appears. The B (E2) value decreases instead of increasing with the increase of  $N_p N_n$ . At the same time the value of  $\gamma_b$  is also reduced to 18° from 25.6° thus, lowering the gap between the  $\gamma_e$  and  $\gamma_b$ . In the next nucleus <sup>174</sup>Hf the difference between  $\gamma_e$  and  $\gamma_b$  vanishes and B (E2) starts increasing again with the increase in  $N_p N_n$ . It continues further in <sup>176</sup>Hf nucleus.

In the above observations it is clear that the erratic value in  $\gamma$  has some role in starting the breakdown in  $N_p N_n$  scheme. The erratic value of  $\gamma$  challenges the internal consistency of rigid triaxial rotor model and also bring breakdown in  $N_p N_n$  scheme. The above systematic repeated again in  $N_p N_n$  scheme where the difference between  $\gamma_e$  and  $\gamma_b$  is large in <sup>180</sup>Hf and the breakdown is followed from <sup>182</sup>Hf.

**Table – I**

Nucl.	N <sub>p</sub> N <sub>n</sub>	E2 <sub>1</sub> <sup>+</sup>	B(E2) ↑	γ <sub>e</sub>	γ <sub>b</sub>
<sup>164</sup> Hf	100	211.0	2.14(18)	19.7	21.8
<sup>166</sup> Hf	120	158.5	3.50(20)	17.2	14.5
<sup>168</sup> Hf	140	124.0	4.30(23)	14.8	17.2
<sup>170</sup> Hf	160	100.8	5.30(12)	12.8	25.7
<sup>172</sup> Hf	180	95.2	4.47(33)	12.5	18.0
<sup>174</sup> Hf	200	90.9	4.88(31)	12.6	12.9
<sup>176</sup> Hf	220	88.4	5.27(10)	10.7	14.9
<sup>178</sup> Hf	200	93.2	4.82(6)	11.2	-
<sup>180</sup> Hf	180	93.3	4.67(12)	11.2	27.0
<sup>182</sup> Hf	160	97.8	5.09*	13.6	<13.7

\*The B (E2) value for <sup>182</sup>Hf is evaluated employing Grodzins [4] relation

$$\frac{A E_{2_1}^+ B(E2; 2_2 \rightarrow 0_1^+)}{Z^2} = (2.5 + 1) \times 10^{-3} \text{ MeV} \cdot e^2 b^2$$

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