# Nonequilibrium steady states in deforming nuclei

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## Introduction

Being a finite quantum system with a certain number of particles, the questions related to applicability of equilibrium and nonequilibrium statistical mechanics for nuclei constitutes a fundamentally significant theme. In this regard, it was Bohr [1]. The re-derivation of all the basic distribution functions (Maxwell-Boltzmann, Fermi-Dirac, Bose-Einstein) for finite systems was made on the basis of complexity of the energy spectrum and by assuming classically chaotic motion of nucleons [2, 3]. A relation between response functions of finite Fermionic systems was established with geometric phase acquired by a nucleon as the shape changes [4]. Subsequent connections with quantum corrections to diffusion [5] in such systems, and, relation with nuclear viscosity tensor [6] has been established. Some years ago, the origin of mass parameters [7] appearing in the large-amplitude collective motion in nuclei was found to originate from the fractal dimension of the path traced by the nuclei in the deformation space as it approaches equilibrium. In this work, we present calculations which demonstrate in unequivocal terms that the equilibration process occurs via the non-adiabatic Landau-Zener-Stückelberg (LZS) transitions.

## Level dynamics and LZS transitions

We consider a nucleus which is undergoing deformation. The energy levels also evolve, as is evident from the Nilsson diagrams. The very fact that one can draw these diagrams implies an assumption of the existence of instantaneous energy spectrum for all deformations which is continuously related to infinitesimally close deformations. This, in turn, implies the assumption of adiabaticity. The transitions between these adiabatic levels would violate the quantum adiabatic theorem of Thirring. Landau, Zener, and Stückelberg showed that the non-adiabatic transition probability between the adjacent levels is given by

$$P_{\mathrm{LZS}} \sim \exp\left[-\frac{2\pi}{\hbar}\frac{\epsilon_{i,i-1}^2}{\left|\frac{d(\epsilon_i-\epsilon_{i-1})}{dt}\right|}\right] \qquad (1)$$

where $\{\epsilon_i\}$ are the single-particle energy levels on the Nilsson diagram. We pose the problem of studying how an initially canonical distribution of a nucleus changes as the system is subjected to deformation. This is a fundamental question - it remains open because a self-consistent calculation taking care of the evolution of energy levels with deformation into the statistical distribution has not been done.

## Evolution of the distribution function, and, appearance of nonequilibrium phenomena

To be concrete, we consider the case of 200 particles which are initially at canonical equilibrium with temperature, T = 5 MeV. We simulate a random motion in the deformation space and assign the energy levels corresponding to a deformation in accordance with the Nilsson diagrams. At a certain deformation, $\eta$, an energy level $\epsilon_i$ will be occupied by $N_i$ particles. As the deformation occurs, there are no transitions among levels except at certain points where the spacing between the adjacent levels becomes much smaller than the mean level spacing at that deformation. At these points, LZS transitions occur. These transitions will eventually alter the canonical distribution, as the system is thrown out of equilibrium. In a strict sense, temperature can no longer be defined [8]. However, we define an "effective temperature", $T_{\mathrm{eff}}$ by assuming that the system is undergoing a quasi-static process with adiabaticity almost everywhere except at the points where LZS transitions occur. $T_{\mathrm{eff}}$

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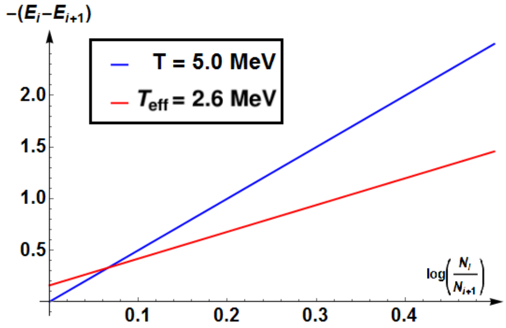


FIG. 1: A random walk in the deformation space is simulated with 1000 steps and the nucleons evolve with the calculated LZS transition probability. At the end of the random walk, a Boltzmann like temperature, $T_{eff}$ has been extracted from the distribution of the particles. The slope of the blue line represents the temperature of the initial Boltzmann distribution which is observed to decrease (red line).

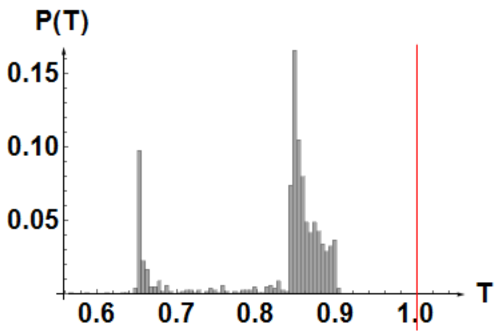


FIG. 2: For an initial temperature of 1 MeV (red line), 1000 realizations of random walk were considered and an effective temperature was deduced. The histogram of these temperatures show a definite decrease in temperature. The two temperatures suggest infact, a temperature gradient.

is shown to decrease for an ensemble of realizations of the shape changes. As seen in Fig. 1, the temperature decreases from 5 MeV to 2.6 MeV.

However, more interestingly, we find that the distribution function ceases to be canonical, as expected. There appears a transport process, hinted by the temperature gradient. This is seen in Fig. 2 below. In this calculation, the initial temperature was taken to be 1 MeV and the number of particles as 200. A large number of realizations have been employed in the simulation, and the temperature profile emerging thereof is plotted here. It is clear that (i) there is cooling, and, (ii) there are two major peaks at temperatures 0.65 MeV and 0.85 MeV. This distribution of temperatures also clarifies the fluctuations clearly.

Thus, as a nucleus de-excites from an excited state, undergoing deformations, it proceeds to nonequilibrium steady states. These states are marked by a production of entropy. We have marked these states by $T_{\rm eff}$. The evidence that a canonically understood temperature is not meaningful is demonstrated in Fig. 2. This study paves the way to incorporate recent advances in nonequilibrium statistical mechanics like Gallavotti-Cohen fluctuation theorems, Jarzynski identity, and Crooks relation in studies on nuclei.

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