Fusion barriers of Th, Pa and U

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Introduction

The fusion reaction is one of the most important reactions in the formation of heavy actinide elements. Various approaches have been suggested to parameterize the fusion barrier heights V_B and barrier position R_B. In the present study, we try to introduce new empirical formulas for barrier characteristics for actinide elements such as Th, Pa and U. This work will introduce simplification in obtaining the fusion barrier positions and heights of these actinide elements. The study of fusion reaction in Th, Pa and U are important in nuclear fuel research.

Theory

By adding the Coulomb potential to the nuclear component, one can compute the total interaction potential, the total interaction potential V(r) between projectile and target nuclei is given by

$$V(r) = V_N(r) + V_C(r) = V_N(r) + \frac{Z_1 Z_2 e^2}{r} \quad (1)$$

Z_1 and Z_2 are atomic numbers of the projectile and target respectively. For nuclear potential V_N, we have used Denisov potential [1].

The fusion barrier has two basic features: one is the barrier position (R_B) and the other is barrier height (V_B). The knowledge of the analytical form of the total interaction potential enables us to determine the exact values of these parameters. Since fusion happens at a distance larger than the touching configuration of colliding pair, the above form of the Coulomb potential is justified. One can extract the barrier height V_B and barrier position R_B using the following conditions

$$\left.\frac{dV(r)}{dr}\right|_{r=R_B} = 0 \quad \text{and} \quad \left.\frac{d^2V(r)}{dr^2}\right|_{r=R_B} \leq 0 \quad (2)$$

According to the Wong approach, the value of $\hbar\omega$ can be determined by [40]

$$\hbar\omega = \hbar\left[\frac{1}{\mu}\left(\frac{d^2V_{tot}(r)}{dr^2}\right)_{r=R_B}\right]^{1/2} \quad (3)$$

Results and discussion

We have studied fusion barrier characteristics of compound nuclei such as Th, Pa and U. That is, we have studied around 200 reactions with different projectile target combinations. After the calculation of fusion barrier heights and positions, we have searched for their parametrization. We first attempted to parametrize fusion barrier position in terms of the radius dependence ($A^{1/3}$). We have plotted the R_B as a function of ($A_1^{1/3}+A_2^{1/3}$) and it is also shown in figure 1. We have fitted the linear equation for R_B interms of ($A_1^{1/3}+A_2^{1/3}$) and it is given as

$$R_B^{para} = 9.7923 + 0.02566 \times \left(A_1^{1/3} + A_2^{1/3}\right) \quad (4)$$

The quality of this linear parametrization is tested by calculating the percentage difference (ΔR_1) between parametrized and exact values and it is shown in Fig. 2. The reduced fusion barrier is $S_B = R_B - C_1 - C_2$, here $C_i = R_i\left[1-(b/R_i)^2\right]$, $R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}$. In Figure 1, we have plotted the reduced fusion barrier positions as a function of $Z_1Z_2/A_1^{1/3}+A_2^{1/3}$. The reduced barrier positions S_B of all projectile target combinations fall on the mean curve that can be parameterized in terms of following equation.

$$S_B^{para} = 4.398564946 - 0.9418533765 \times ln\left[\frac{z_1 z_2}{A_1^{1/3}+A_2^{1/3}}\right] \quad (5)$$

The analytical parametrized fusion barrier positions become

$$R_B^{par} = S_B^{par} + C_1 + C_2 \quad (6)$$

The quality of our parametrized fusion positions can be judged by analyzing the percentage deviation (ΔR_2) and it is also shown in figure 2.

Figure 1: Variation of fusion barrier positions R_B (fm) and reduced fusion barrier positions S_B (fm)

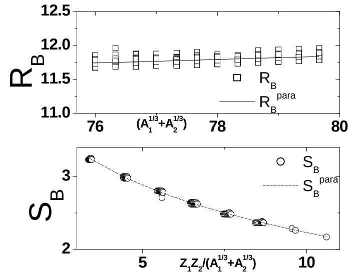
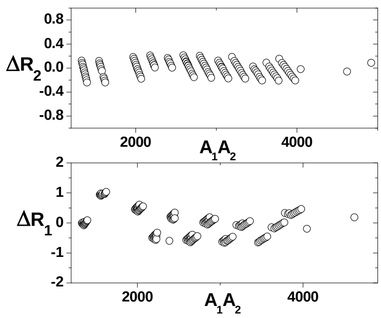


Figure 2: The percentage difference ΔR_B(%) for parameterization of fusion barrier position as a function of the product of charges of colliding pair A_1A_2.



The percentage difference ΔR_B(%) of empirical formula for barrier positions R_B based on the concept of reduced barrier position is almost less than 0.4%. We have parametrized the fusion barrier heights V_B as a function of $\left(Z_1Z_2/R_B^{par}\right)\left(1-1/R_B^{par}\right)$, similar to the one reported by earlier workers [2-3]. The first part is the Coulomb contribution whereas the second part is the reduction due to the nuclear potential. We have plotted the V_B as a function of $\left(Z_1Z_2/R_B^{par}\right)\left(1-1/R_B^{par}\right)$ and it is also shown in figure 3. The fusion barrier heights V_B of all projectile target combinations fall on the mean curve that can be parameterized in terms of following equation.

$$V_B^{para} = 0.10339 + 1.46627 \times \left[\frac{Z_1Z_2}{R_B^{para}}\left(1-\frac{1}{R_B^{para}}\right)\right] \quad (7)$$

The quality of our analytical parametrization is tested in Figure 4, where percentage difference between parametrized and exact values of fusion barrier heights V_B are shown. The percentage difference ΔV_B(%) of empirical formula for barrier height V_B is almost less than 0.2%. After the calculation of inverted parabola of fusion barrier, we have searched for their parametrization. We have plotted inverted parabola ħω as a function of $\left(Z_1Z_2/R_B^{par}\right)\left(1-1/R_B^{par}\right)$ and it is shown in figure 3. The inverted parabola of fusion barrier for all projectile target combinations is fall on the mean curve that can be parameterized in terms of following equation.

$$\hbar\omega = 4.286808925 - 5.305928548 \times 10^{-4} \times \left[\frac{Z_1Z_2}{R_B^{para}}\left(1-\frac{1}{R_B^{para}}\right)\right] \quad (8)$$

The percentage difference $\Delta\hbar\omega$(%) is also shown in figure 3. The presented empirical formulae for R_B, V_B and ħω are valid for projectile, target and compound nuclei of different atomic and mass number ranges such as $3 \le Z_1 \le 18$, $72 \le Z_2 \le 82$, $6 \le A_1 \le 40$, $165 \le A_2 \le 233$, $90 \le Z \le 92$ and $205 \le A \le 239$.

Figure 3: Fusion barrier heights V_B (fm) and ħω as a function of $\left(Z_1Z_2/R_B^{par}\right)\left(1-1/R_B^{par}\right)$

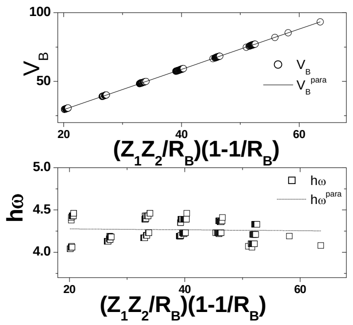
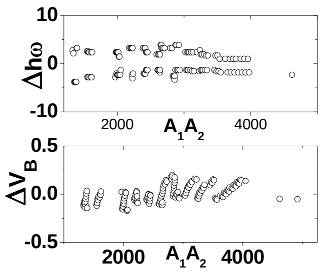


Figure 4: The percentage difference $\Delta\hbar\omega$ (%) and ΔV_B as a function of the product of mass numbers of colliding pair A_1A_2.



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