Analytical calculation of Thermodynamical properties of paired even-even nuclei

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Introduction

Nuclear super-fluidity has an important role in describing the nuclear structure and properties of nuclear systems. The odd-even nuclear mass differences and the energy gap in even-even nuclei are well known signatures of pairing effect. The original idea of pairing phenomenon in nuclei was proposed by Bohr, Mottleson and Pines [1], just one year after the publication of the theory of superconductivity by Bardeen, Cooper and Schrieffer (BCS).

Using semiclassical trace formulae, we have already understood the melting of shell effects in magic nuclei [2]. In this work, we have calculated the thermodynamical quantities: excitation energy (E), entropy (S) and nuclear level density (ρ) for even-even Calcium isotopes including shell and pairing effects at zero and finite temperatures.

Theory

The single-particle level density for a harmonic oscillator potential is given as [3]:

$$g(E) = \frac{1}{2(\hbar\omega)^3}\left[E^2 - \frac{1}{4}(\hbar\omega)^3\right]\left\{1 + 2\,Cos\left(\frac{2\pi E}{\hbar\omega}\right)\right\}. \quad (1)$$

Here, the first term corresponds to the smooth part ($\tilde{g}(E)$), while the second term represents the oscillatory part ($\delta g(E)$) of the level density. Using average spacing between the levels of the nucleus as :

$$\hbar\omega(n,p) = \frac{41}{A^{\frac{1}{3}}}\left(1 \pm \frac{(N-Z)}{A}\right)^{\frac{1}{3}} MeV \quad (2)$$

We can determine the neutron and proton Fermi energies from the conservation of particle number as:

$$N, Z = \int_0^{E_F^{(n,p)}} g(E)dE \quad (3)$$

Then, the excitation energy E, entropy S and nuclear level density ρ are determined using Bethe's formulae [4]:

$$E = \frac{\pi^2}{6}(g_n + g_p)T^2 \quad (4)$$

$$S = \frac{\pi^2}{3}(g_n + g_p)T \quad (5)$$

$$\rho = \frac{6^{\frac{1}{4}}}{12} g_0 \left(\frac{{g_0}^2}{4\, g_n\left(E_F^{(n)}\right) g_p\left(E_F^{(p)}\right)}\right)^{\frac{1}{2}} (g_0 E)^{-\frac{5}{4}}$$

$$\times \exp\left\{2\left(\frac{\pi^2}{6} g_0 E\right)^{\frac{1}{2}}\right\} \quad (6)$$

Here, $g_0 = (g_n + g_p)$ is the total single particle level density evaluated at Fermi energies of each nucleon.

To include the pairing effects, we need to determine the odd-even nuclear mass differences Δ_n, Δ_p which are as given below [5]:

$$\Delta_n = \frac{1}{4}[B(Z, N-2) - 3B(Z, N-1) + 3B(Z, N) - B(Z, N+1)] \quad (7)$$

$$\Delta_p = \frac{1}{4}[B(Z-2, N) - 3B(Z-1, N) + 3B(Z, N) - B(Z+1, N)] \quad (8)$$

The pairing correlation energy δP is calculated exploiting pairing gap [5]:

$$\delta P_i = -\frac{1}{2} g_i\left(E_F^{(i)}\right)\Delta_i^2 \quad \text{with } i = n, p \quad (9)$$

However, the critical temperature T_{ci}, which is the temperature at which pairing gap disappears as given by the BCS approximation :

$$T_{ci} = \frac{2\Delta_i}{3.50} \quad (10)$$

Calculations of thermodynamic quantities

The temperature dependence of the excitation energy E_i for n, p each is taken as [6]:

$$\left.\begin{aligned} E_i(T) &= a_i T^2 - \delta P_i \frac{T^2}{T_{ci}^2} \quad For\ T \leq T_{ci} \\ E_i(T) &= a_i T^2 - \delta P_i\ For\ T \geq T_{ci} \end{aligned}\right\} \quad (11)$$

where, a_i is level density parameter. The critical excitation energy $E_c^{(i)}$ for the total system at T_{ci} is given by:

$$E_c^{(i)} = E_p(T_{ci}) + E_n(T_{ci}) \quad (12)$$

The effect of pairing both above and below $E_c^{(i)}$ can be incorporated with the help of continuous differentiable translation function Q [6]:

$$Q = R_p \delta P_p + R_n \delta P_n \quad (13)$$

where, $R_i = \begin{cases} \frac{2E}{E_c^{(i)}} - \left(\frac{E}{E_c^{(i)}}\right)^2 & For\ E < E_c^{(i)} \\ 1 & For\ E \geq E_c^{(i)} \end{cases}$

With the inclusion of pairing effects, E, S and ρ transform as:

$$E_Q = E + Q \quad (14)$$

$$S_Q = \sqrt{\frac{2\pi^2}{3}\left(g_n + g_p\right) E_Q} \quad (15)$$

$$\rho_Q = \frac{6^{\frac{1}{4}}}{12} g_0 \left(\frac{g_0{}^2}{4\, g_n\left(E_F^{(n)}\right) g_p\left(E_F^{(p)}\right)}\right)^{\frac{1}{2}} \left(g_0 E_Q\right)^{-\frac{5}{4}}$$

$$\times \exp\left\{2\left(\frac{\pi^2}{6} g_0 E_Q\right)^{\frac{1}{2}}\right\} \quad (16)$$

Table1: Values of proton and neutron pairing gaps Δ_p, Δ_n and critical temperatures T_{cp}, T_{cn} for Calcium nuclei.

Nuclei	Δ_p (MeV)	Δ_n (MeV)	T_{cp} (MeV)	T_{cn} (MeV)
$_{20}Ca^{42}$	1.563	1.333	0.893	0.762
$_{20}Ca^{44}$	1.606	1.383	0.918	0.790
$_{20}Ca^{46}$	1.581	1.221	0.904	0.698
$_{20}Ca^{48}$	1.742	1.496	0.995	0.855
$_{20}Ca^{50}$	1.824	0.551	1.042	0.315

Results and Discussions

The results of E, S and $Log_{10}[\rho]$ for $_{20}Ca^{42}$ are shown in the Fig.1, Fig.2 and Fig.3 respectively. Black solid curve in each figure represents the result of inclusion of shell effects along with pairing effects. Whereas the grey solid curve shows the results including pairing effects only.

It is seen that above T_{cn}, T_{cp} (marked in fig.1) nucleonic pairs break up, thus the curve (solid curve) overlaps with the no pairing curve (dotted curve).

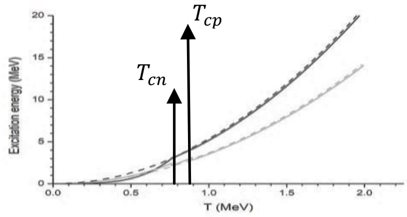

Fig.1: Plot of excitation energy (E) w.r.t to temperature (T).

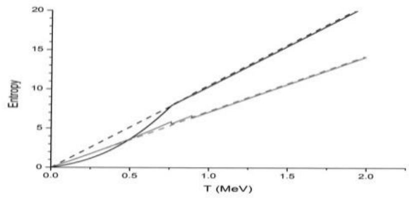

Fig.2: Plot for the entropy (S) w.r.t to temperature (T).

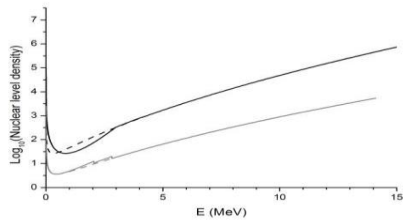

Fig.3: Plot of Log_{10} (Nuclear level density) and the excitation energy for Ca-isotopes.

A significant difference observed at low energies indicates the importance of pairing correlations in interpretation of thermodynamical properties and hence its application in reaction theory and the astrophysics.

References

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