Nuclear matrix elements calculations of neutrinoless double beta decay of $^{48}_{20}Ca$ in pure shell model

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Introduction

Neutrinoless double beta decay is very rare lepton number violating second order weak process where two neutrons inside some even even nuclei converted into two protons and two electrons [1]. Neutrino comes as a virtual state. It proves that neutrinos are Majorana particle rather than Dirac particle and also predict the absolute neutrino mass. But even after 70 years of its first prediction, this rare process is still unobserved. Problem lies in predicting the proper half life which in turn depends upon calculations of nuclear matrix elements(NME) with accuracy. Limitations of understanding complex nuclear structure leads to the uncertainties in NME. In this paper we will discuss those theoretical uncertainties of NME calculations and calculate NME for the neutrinoless double beta decay $^{48}_{20}Ca \rightarrow^{48}_{22} Ti + e^- + e^-$ using pure shell model.

Theoretical Framework

We considered here the most dominating light left handed Majorana neutrino exchange mechanism of neutrinoless double beta decay.

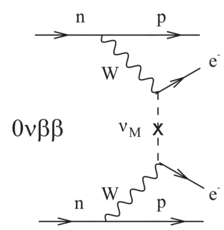

FIG. 1: Light neutrino exchange mechanism of neutrinoless double beta decay

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Decay rate for this process can be written as

$$\Gamma^{0\nu} = \frac{1}{T^{0\nu}_{1/2}} = G^{0\nu}(Q, Z)|m_{\beta\beta}|^2|M^{0\nu}|^2 \quad (1)$$

$G^{0\nu}(Q, Z)$ is phase space factor and $m_{\beta\beta}$ is absolute mass of Majorana neutrino. $M^{0\nu}$ is nuclear matrix elements and is given by

$$M^{0\nu} = M^{0\nu}_{GT} - \frac{g_V^2}{g_A^2} M^{0\nu}_F + M^{0\nu}_T \quad (2)$$

Each part of nuclear matrix element can be written as [2]

$$M^{0\nu}_\alpha = \sum_{j_p j_{p'} j_n j_{n'} J} TBTD(j_p, j_{p'}, j_n, j_{n'}, J^\pi_i, J^\pi_f) \times < j_p j_{p'}, JT, 0^+_i | \tau^+_1 \tau^+_2 O^{0\nu}_\alpha | j_n j_{n'}, JT, 0^+_f >_A \quad (3)$$

Here $\alpha = F, GT, T$ and A stands for anti-symmetric. Transition operators $O^{0\nu}_\alpha$ are written as $O^{0\nu}_{GT} = \vec{\sigma}_1.\vec{\sigma}_2 H_{GT}(r)$, $O^{0\nu}_F = H_F(r)$, $O^{0\nu}_T = (3(\vec{\sigma}_1.\hat{r})(\vec{\sigma}_2.\hat{r}) - \vec{\sigma}_1.\vec{\sigma}_2)H_T(r)$. Using Talmi Moshinisky transformations we transform the problem from individual coordinate system of nucleons to center of mass(c.o.m) and relative coordinate system. Then we need to calculate neutrino potential integral of the form $< nl|H_\alpha(r)|nl >= \int_0^\infty R_{nl}R_{nl}r^2 * H_\alpha(r)dr$. Here R_{nl} is radial wave function of 3D harmonic oscillator basis in nuclear shell model. r is inter nucleon distance. $H_\alpha(r)$ is neutrino potential and written as a integral of neutrino momentum q [3]
$H_\alpha(r) = \frac{2R}{\pi} \int_0^\infty \frac{q \times j_{0,2}(qr) \times f^2_{FNS}(q^2) \times g^\alpha_{HOT}(q^2) dq}{q + \overline{E}}$

It is the most important part of nuclear matrix elements containing various sources of uncertainties. First such uncertainty source is finite nucleon size. Nucleons are made up of quarks and it is taken care by a form factor $f_{FNS}(q^2) = \frac{1}{(1+\frac{q^2}{M_A})^2}$. In addition to $V-A$ coupling there are higher order coupling like weak magnetism and pseudoscalar at nucleon level decay of neutrinoless double beta decay and are taken care by higher order coupling term

$$g^F_{HOT} = 1 \quad (4)$$

$$g^{GT}_{HOT} = 1 - \frac{2}{3}\frac{q^2}{q^2+m_\pi^2} + \frac{1}{3}\left(\frac{q^2}{q^2+m_\pi}\right)^2 + \frac{2}{3}\left(\frac{g_V(q^2)}{g_A(q^2)}\right)^2 \times \frac{(\mu_p-\mu_n)^2}{4m_p^2} \times q^2 \quad (5)$$

$$g^T_{HOT} = \frac{2}{3}\frac{q^2}{q^2+m_\pi^2} - \frac{1}{3}\left(\frac{q^2}{q^2+m_\pi}\right)^2 + \frac{1}{3}\left(\frac{g_V(q^2)}{g_A(q^2)}\right)^2 \frac{(\mu_p-\mu_n)^2}{4m_p^2} \times q^2 \quad (6)$$

Another uncertainty term is short range correlation. Force between nucleons at very short distance is strongly repulsive. It is taken care by Jastrow function $R_{nl} \to f_{Jastrow}R_{nl}$, where $f_{Jastrow}(r) = 1 - ce^{-ar^2}(1-br^2)$
a, b, c are constant parameter.

Calculations of Nuclear Matrix Elements

We did our calculations for neutrinoless double beta decay of

$$^{48}_{20}Ca \to^{48}_{22} Ti + e^- + e^-.$$

In pure shell model all eight active neutrons of $^{48}_{20}Ca$, active two protons of $^{48}_{22}Ti$ are in $f_{7/2}$ orbit. Using above formalism, the important part neutrino potential integral for both without SRC and with SRC were calculated and few results shown in table I and II respectively. Tensor part in our calculations is negligible.

Calculation of Talmi Moshinisky bracket was done using the formalism given in [4]. Spin part of NME was calculated using standard method. TBTD is calculated in second quantization in terms of creation and annihilation operator. For our calculation we used the results of [5]. Final calculated value of nuclear matrix elements are given in the following table.

TABLE I: Table of calculated values of neutrino potential integral without SRC

n,l	$< nl\|H_F(r)\|nl >$	$< nl\|H_{GT}(r)\|nl >$
3,0	0.820	1.281
2,0	0.893	1.395
0,0	1.203	1.880
0,1	0.623	0.974

TABLE II: Table of calculated values of neutrino potential integral with SRC

n,l	$< nl\|H_F(r)\|nl >_{SRC}$	$< nl\|H_{GT}(r)\|nl >_{SRC}$
3,0	0.553	0.864
2,0	0.665	1.039
0,0	1.119	1.749
0,1	0.641	1.003

TABLE III: Final value of nuclear matrix elements

$M^{0\nu}_F$	$M^{0\nu}_{GT}$	$(M^{0\nu}_F)_{SRC}$	$(M^{0\nu}_{GT})_{SRC}$
-1.302	0.678	-1.010	0.526

Summary

We have calculated nuclear matrix elements for the decay $^{48}_{20}Ca \to^{48}_{22} Ti + e^- + e^-$ in pure shell model. Calculations was done with all sources of uncertainties.

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