A new approach to understand nuclear stability and binding energy

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Introduction

In our earlier contribution [1,2], we made a very simple attempt in implementing the strong coupling constant (α_s) for understanding nuclear stability and binding energy [3]. In this contribution, we developed another way of implementing (α_s) along with neutron-proton mass difference.

Basic conceptual thoughts

We would like to suggest that,

1) Strong coupling constant [2] seems to play an interesting role in nuclear binding energy scheme.
2) As nuclear size is increasing, it is natural to expect a very small reduction in α_s. Based on this idea, we imagine that, for Z=2, $\alpha_s \approx 0.1175$ and for Z=92, $\alpha_s \approx 0.114$.
3) Neutron-proton mass difference and electron mass seem to play a crucial role in understanding the beta stability line and nuclear binding energy.
4) Proton number plays an interesting 'proportionality' role in understanding nuclear binding energy at stable mass number.
5) For $(Z \geq 30)$, close to the beta stability line, nuclear binding energy and Z protons kinetic energy seem to be approximately equal in magnitude [1,4,5].
6) Z=30 seems to play a characteristic role in nuclear binding energy scheme.
7) Ratio of neutron number and stable neutron number seems to play a crucial role in estimating the isotopic binding energy of Z.
8) Need of considering arbitrary energy coefficients can be reviewed.

To understand the beta stability line

With reference to neutron, proton and electron rest masses [4], it is interesting to note that,

$$\left.\begin{array}{l}\exp\left[\left(m_n c^2 - m_p c^2\right)/m_e c^2\right] \cong (4\pi) \\ \rightarrow \left[\left(m_n c^2 - m_p c^2\right)/m_e c^2\right] \cong \ln(4\pi)\end{array}\right\} \quad (1)$$

Based on this observation and with reference to beta stability line, if A_s and N_s represent stable mass number and stable neutron number of Z respectively,

$$\left.\begin{array}{l}A_s \cong 2Z + [Z/4\pi]^2 \cong 2Z + 0.0063326Z^2 \\ N_s \cong Z + [Z/4\pi]^2 \cong Z + 0.0063326Z^2\end{array}\right\} \quad (2)$$

These relations can be compared with the computational relation pertaining to isotonic shift and drip lines proposed in reference [3], $N_s = 0.968051Z + 0.00658803Z^2$. With even-odd corrections much better correlations can be observed. See the first two columns of table-1.

Nuclear binding energy at stable mass numbers

At stable mass numbers of Z, nuclear binding energy can be expressed by the following empirical relation.

$$(B)_{A_s} \cong kZ\left(m_n - m_p\right)c^2 \quad (3)$$

$$\left\{\begin{array}{l}\text{where,} \\ \text{If } (Z<30),\ \text{coefficient, } k \cong \left[\dfrac{1}{(\alpha_s)_Z} + \sqrt{Z} + 1\right] \\ \text{If } (Z \geq 30),\ k \cong \left[\dfrac{1}{(\alpha_s)_Z} + \sqrt{30} + 1\right] \\ \text{and } (\alpha_s)_Z \approx 0.1175 - (0.006333)^2 Z\end{array}\right.$$

Binding energy below and above the stable mass numbers

With reference to the semi empirical mass formula, approximately, below and above the stable mass number, nuclear binding energy can be expressed in the following form.

$$\left.\begin{array}{l} B_{(Z,A)} \cong \left(\dfrac{A-Z}{A_s-Z}\right)^p kZ \times 1.2933 \text{ MeV} \\ \text{if } (A < A_s),\ p \cong (2/3);\ \text{if } (A > A_s),\ p \cong (1/2) \end{array}\right\} \quad (4)$$

See the following table1 for data comparison.

Table 1. To Fit the measured binding energy

Z	A with even odd correction	Binding energy from SEMF (MeV)	Estimated binding energy (MeV)
2	4	28.296	28.3
6	12	92.162	92.9
10	20	160.645	164.3
20	42	361.896	363.2
26	56	492.258	493.8
36	80	695.434	702.7
44	100	861.928	860.3
50	116	988.684	978.7
60	142	1185.142	1176.8
70	170	1378.13	1375.7
82	206	1622.325	1615.5
92	238	1801.69	1816.1

See the following figure 1 pertaining to binding energy of isotopes of Z=50.

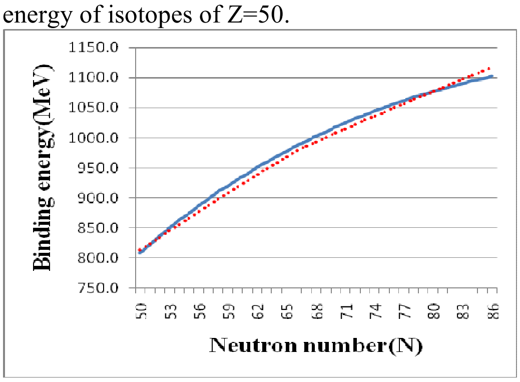

Figure 1: Comparative binding energy curves of isotopes of Z=50

Solid (blue) curve indicates the SEMF binding energy and dotted (red) curve indicates the estimated binding energy. It may be noted that, curve fitting mainly depends on the accuracy of the assigned binding energy for the estimated stable isotope of Z=50.

Discussion

1. $Z=53$ is estimated to be stable at $A_s = 123$ with an assigned binding energy of 1038.6 MeV. Actually, it is stable at $A_s = 127$. From relation (4), estimated binding energy of ${}_{50}I^{127}$ is 1067.9 MeV and its actual binding energy is 1072.57 MeV.
2. We are working on understanding the physical significance of the coefficients, $k \cong \left[\frac{1}{(\alpha_s)_Z} + \sqrt{Z} + 1\right]$ and $k \cong \left[\frac{1}{(\alpha_s)_Z} + \sqrt{30} + 1\right]$.
3. On ignoring the strong coupling constant, estimated binding energy of deuteron seems to be 2*1.2933=2.587 MeV and can be compared with the experimental value of 2.225 MeV.

Conclusion

We proposed a new method for understanding nuclear stability and binding energy in terms of strong interaction and nucleon mass difference. Our proposal can be recommended for further study at academic level.

References

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