Nuclear Radius Correction to Bethe-Weizsacker Mass Formula

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Introduction

Bethe-Weizsacker semi-empirical mass formula (WF) [1][2] was put forward to explain Binding energy (E_0) data of stable and near-stable nuclei. For a nucleus with A nucleons, Z protons and N neutrons the formula [3] is given as:

$$E_0(A,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \frac{\delta}{A^{1/2}} \quad (1)$$

$$\begin{aligned} \delta &= +a_P \quad \text{if N is even and Z is even} \\ &= 0 \quad \text{if A is odd} \\ &= -a_P \quad \text{if N is odd and Z is odd} \end{aligned}$$

where a_V,a_S,a_C and a_P are fit parameters. The formula assumes the relation:

$$R = r_0 A^{1/3} \quad \text{or} \quad A = \left(\frac{R}{r_0}\right)^3 \quad (2)$$

for nuclear radius (R) and A. A number of improvements over the crude radius formula given in Eq.(2), exists [4][5][6]. In this article we propose to modify WF to include effects of deviation of nuclear radius from $R = r_0 A^{1/3}$ behaviour. WF is modified by first explicitly writing the first three (classical) terms of WF in terms of nuclear radius using Eq.(2) and then inserting an improved nuclear radius formula. Modified mass formula (MF) is thus sensitive to details of nuclear radius dependence on A and Z. Different improved nuclear radius formulae are employed to test the validity of our modification. Comparison between MF and WF fits to stable nuclei and predictions for unstable nuclei with half-life $\tau > 1$ year, are presented.

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Modification to Bethe-Weizsacker Mass formula

Replacing A with R, via Eq.(2), in Eq. (1) and employing different experimentally consistent improved nuclear radius formulae

$$R_1 = r_0(A^{1/3} + \frac{a_1}{A^{1/3}}) = r_0\phi_1 \quad (3)$$

$$R_2 = r_0(A^{1/3} + \frac{a_1}{A^{1/3}} - a_2) = r_0\phi_2 \quad (4)$$

$$R_3 = r_0(A^{1/3} + \frac{a_1}{A^{1/3}} + \frac{a_2}{A}) = r_0\phi_3 \quad (5)$$

$$R_4 = r_0(A^{1/3} + \frac{a_1}{A} + a_2\frac{(A-2Z)}{A^{2/3}}) = r_0\phi_4 \quad (6)$$

we get,

$$E_1 = a'_V\phi_1^3 - a'_S\phi_1^2 - a'_C\frac{Z(Z-1)}{\phi_1} - a_A\frac{(A-2Z)^2}{A} + \frac{\delta}{A^{1/2}} \quad (7)$$

$$E_2 = a'_V\phi_2^3 - a'_S\phi_2^2 - a'_C\frac{Z(Z-1)}{\phi_2} - a_A\frac{(A-2Z)^2}{A} + \frac{\delta}{A^{1/2}} \quad (8)$$

$$E_3 = a'_V\phi_3^3 - a'_S\phi_3^2 - a'_C\frac{Z(Z-1)}{\phi_3} - a_A\frac{(A-2Z)^2}{A} + \frac{\delta}{A^{1/2}} \quad (9)$$

$$E_4 = a'_V\phi_4^3 - a'_S\phi_4^2 - a'_C\frac{Z(Z-1)}{\phi_4} - a_A\frac{(A-2Z)^2}{A} + \frac{\delta}{A^{1/2}} \quad (10)$$

Equation (7) has one additional parameter a_1, whereas Eqs.(8)–(10) have two additional parameters a_1 and a_2.
Each of Equations (7)–(10) is one MF for one choice of improved radius formulae. Thus Eq.(7) (MF_1), Eq.(8) (MF_2), Eq.(9) (MF_3) and Eq.(10) (MF_4) are modified formulae for

radius formula R_1, R_2, R_3 and R_4 respectively. Error comparison between WF and our modified formulae, MF_1 are shown below.

Fig 1: WF fit to Binding Energy for all stable nuclei

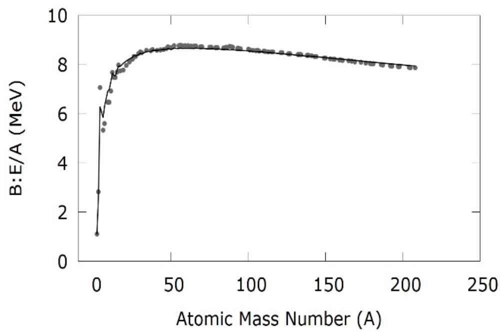

Fig 2: MF_1 fit to Binding Energy for all stable nuclei

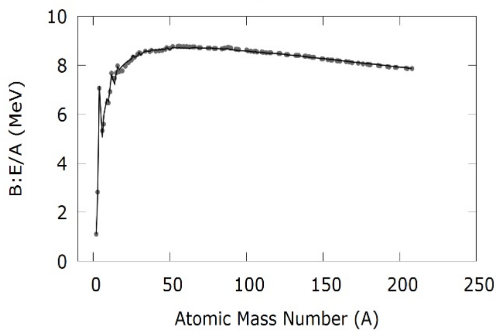

TABLE I: Error comparisons between WF and MFs fits to B:E data for stable nuclei.

	%Error				
	WF	MF_1	MF_2	MF_3	MF_4
$N, Z \geq 1$	0.855	0.324	0.312	0.324	0.318
$N, Z > 8$	0.214	0.216	0.214	0.214	0.192

TABLE II: Error comparisons between WF and MFs B:E predictions for $\tau > 1yr$.

	%Error				
	WF	MF_1	MF_2	MF_3	MF_4
$N, Z \geq 1$	1.25	0.336	0.267	0.336	0.241
$N, Z > 8$	0.302	0.253	0.302	0.302	0.282

Results and Discussions

From Tables I & II, we observe that MFs give overall better fits and predictions than WF. Both MF and WF give similar accuracies for N, $Z > 8$. Both description and predictions with MF show 50-70% reduction in error percentage as compared to WF. MF_1 is found to be particular optimum MF among other MFs, as it has relatively simple form, gives accuracies very similar to other MFs and has only one additional parameter as compared to WF.

Conclusions and Suggestions

Applying radius correction to volume, surface and coulomb energy terms in Bethe-Weizsacker Mass Formula leads to better understanding of binding energy data, especially for lighter nuclei. We have tested our modification on simple Bethe-Weizsacker mass formula. Effect of similar modification to modern semi-empirical mass formula needs further investigation.

References

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