Comparative study of Properties of stable(^{4}He) and weakly bound(^{6}He) Helium isotopes using Skyrme Pairing Force-SKP Functional

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In past, several attempts have been made to understand the relationship between effective nucleon-nucleon interactions and observed static and dynamic properties of nuclei such as binding energies, root mean square (r.m.s.) radii, saturation properties of nuclear forces, densities of nucleons, symmetry energy and incompressibility of nuclear matter at given density. For instance, authors in Refs.[1-3] have used Tabakin's potential to solve the Hartree-Fock problems up to second and third order correction terms to estimate some of nuclear properties. But such calculations were unable to explain the observed densities and radii of nuclei in a consistent way. The serious difficulty with these calculations was the improper saturation properties of Tabakin's potential. Davies and his group [4] have made calculations within the framework of Brueckner Hartree-Fock method by adopting Reid's soft core potential but still unable to reproduce the nuclear radii and binding energies of nuclei. Such discrepancies can be correlated with the higher order terms in the expansion of Reid's soft core form of potential which was not suitable for description of nucleon-nucleon interactions.

A more realistic approach to overcome aforementioned difficulties is the use of Hartree-Fock method with effective skyrme interactions wherein one can circumvent completely the problem of addition of higher order terms. The skyrme interactions are density dependent interactions and provide satisfactory description of the radii, binding energies and single particle energies of doubly closed shell nuclei. The Hartree-Fock method with Skyrme interaction thus becomes the most widely utilized approach to analyse the nuclear structure and related properties.

In its original form Skyrme's interaction can be written as a potential [5].

$$V = \sum_{i<j} v_{ij}^{(2)} + \sum_{i<j<k} v_{ijk}^{(3)}$$

with, $v_{ij}^{(2)}$ is a two body terms and $v_{ijk}^{(3)}$ is a three body term. The two body terms and three body terms were modified by different authors by fitting the large set of experimental data available in literature [6-7]. In configuration space, the two body interaction terms is defined as

$$\begin{aligned} v_{ij}^{(2)} &= t_0(1+x_0P^{\sigma})\delta(\vec{r}_1-\vec{r}_2) \\ &+\frac{1}{2}t_1(1+x_1P^{\sigma})\left[\delta(\vec{r}_1-\vec{r}_2)\vec{k}^2+\vec{k}'^2\delta(\vec{r}_1-\vec{r}_2)\right] \\ &+t_2(1+x_2P^{\sigma})\vec{k}'\delta(\vec{r}_1-\vec{r}_2)\vec{k} \\ &+iW_0(\vec{\sigma}_1+\vec{\sigma}_2)\bullet\vec{k}'\times\delta(\vec{r}_1-\vec{r}_2)\vec{k} \end{aligned}$$

with, $\vec{k}$ and $\vec{k}'$ as relative wave vectors of two nucleons respectively. P^{σ} is the spin-exchange operator and $\vec{\sigma}$ are Pauli spin matrices. W_0 is spin-orbit term. The three body term assumes a zero range force which is equivalent to a two body density dependent interactions [1] and is defined as

$$v_{ijk}^{(3)} = \frac{1}{6}t_3(1+x_3P^{\sigma})\left[\delta(\vec{r}_1-\vec{r}_2)\right]\rho^{\alpha}$$

Such term provides a simple phenomenological representation of many body effects. The simple structure of Skyrme force allows one to express Hamiltonian density for a system described by a Slater determinant as an algebraic sum function of the nuclear and kinetic energy densities. In this approach, the total Hamiltonian H_T can be expressed as the integral of the density functional [6,7] as given below.

$$H_T = \langle \Psi | H | \Psi \rangle = \int H d^3 r$$

with,

$$H = H_{kin} + H_0 + H_{density} + H_{eff} + H_{fin} + H_{so} + H_{sg} + H_{Coul}$$

and various terms have their usual meanings.

Besides these, pairing correlations have been known to influence nuclear structure and reaction dynamics of spherical and deformed nuclei and hence must be entertained in the theoretical description. Gogny and his collaborators [8] within the framework of Hatree-Fock bogolyubov (HFB) theory developed an effective interaction appropriate for description of the mean field and pairing correlations. The Hartree-Fock method with effective Skyrme interaction, wherein one can work in coordinate space and properly handle the particle continuum states in nuclei close to drip lines, is another simple alternative way to include the pairing effects.

Thus in the present work, we have used the Skyrme Pairing (SkP) functional [8] to study properties of stable and weakly bound isotopes of He. The results of the present calculations are listed in Table1.

Table 1: Comparison of stable ^{4}He and weakly bound Borromean ^{6}He nuclei properties.

Radius (*fm*)		
Property	**^{4}He**	**^{6}He**
Proton RMS Radius	1.956656	1.972735
Neutron RMS Radius	1.948751	2.389867
Total RMS Radius	1.952708	2.259396
Charge Radius	2.124713	2.112223
Neutron Skin	-0.007906	0.417133
Pairing Energy (*MeV*)		
Binding Energy	-29.983718	-36.159424
Proton Fermi Energy	-10.558154	-18.715632
Neutron Fermi Enrgy	-11.478886	-4.272001
Proton Pairing Gap	5.597249	5.015006
Neutron Pairing Gap	5.649244	3.725048

One can easily extract the following meaningful and important conclusions from the results given in Table1. For weakly bound ^{6}He nucleus, the neutron r.m.s. radius as well as the total r.m.s. radius is much larger than its stable counterpart ^{4}He and confirms the fact that ^{6}He has neutron halo structure.

In addition, the pairing energies of protons and neutrons are treated separately for both nuclei, since neutron rich nuclei would have higher single particle energy of the last filled neutron than the one for protons, so pairing gaps decreases with increasing neutron excess which can be seen from neutron pairing gap value of ^{6}He from table. Due to excess neutrons and smaller neutron pairing gap ^{6}He has a neutrons Borromean structure.

In future our efforts will be focussed to use nuclear energy density functional to unravel various structural properties of weakly bound stable and unstable nuclei and hence to understand the reaction dynamics for reactions induced by these nuclei.

References

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