Searching for a universal correlation among symmetry energy parameters

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Introduction
Symmetry energy parameters have profound implications in terrestrial nuclear physics as well as in astrophysics and cosmology, as they carry the signatures of isovector part of the nuclear interaction. Symmetry energy coefficient can be defined as,

\[ C_2(\rho) = \frac{1}{2} \left( \frac{\partial^2 e(\rho, \delta)}{\partial \delta^2} \right)_{\delta=0}, \tag{1} \]

where \( e(\rho, \delta) \) is the energy per nucleon of infinite nuclear matter (NM) with density \( \rho = \rho_n + \rho_p \) and isospin asymmetry \( \delta = \frac{\rho_n - \rho_p}{\rho} \), where, \( \rho_n (\rho_p) \) is the neutron (proton) density. The symmetry energy coefficient can be expanded around the saturation density \( \rho_0 = (0.16 \text{ fm}^{-3}) \) as,

\[ C_2(\rho) \simeq C_2^0 \rho_0 + L_0 \epsilon_0 + \frac{1}{2} K_{\text{sym}}^0 \rho_0^2 + \frac{1}{6} Q_{\text{sym}}^0 \rho_0^3. \tag{2} \]

Here, \( C_2^0 \) is symmetry energy coefficient at \( \rho_0, L_0 \) is slope parameter related to the first derivative of \( C_2(\rho) \) with respect to density \( \rho \). Similarly, \( K_{\text{sym}}^0 \) and \( Q_{\text{sym}}^0 \) are connected to the second and third derivative of \( C_2(\rho) \) with respect to \( \rho \) called the curvature parameter and the skewness parameter, respectively.

\( C_2^0 \) is known in quite tight bound (32±3 MeV) from precisely known experimental data on binding energies of nuclei. However, \( L_0 \) is not that well-constrained. From different experimental constraints the present accepted range of \( L_0 \) is 35 MeV < \( L_0 < 85 \) MeV. The uncertainty becomes even larger when one tries to constrain the value of \( K_{\text{sym}}^0 \) or \( Q_{\text{sym}}^0 \). Till date there is no constraint from experiment to determine the values of \( K_{\text{sym}}^0 \) or \( Q_{\text{sym}}^0 \) precisely. Across various relativistic and non-relativistic mean-field models, the values of \( K_{\text{sym}}^0 \) and \( Q_{\text{sym}}^0 \) lie over a very wide range -700 MeV < \( K_{\text{sym}}^0 < 400 \) MeV and -800 MeV < \( Q_{\text{sym}}^0 < 1500 \) MeV [1, 2]. Using a representative set of relativistic and non-relativistic mean-field models an empirical linear correlation between \( L_0 \) and \( K_{\text{sym}}^0 \) is suggested [3, 4]. However, the results seem to be dependent on the choice of set of models. We address this problem in this contribution by deriving analytical relations among different symmetry energy elements with in a simplistic model and validate those relations by using 500 relativistic and non-relativistic mean-field models used in the literature.

Theoretical Model
Starting with some basic equations of statistical mechanics and using few reasonable assumptions we obtained an energy density functional (EDF) of NM with density \( \rho \) and asymmetry \( \delta \) [5]. Using this this EDF expressions for \( K_{\text{sym}}^0 \) and \( Q_{\text{sym}}^0 \) can be written as,

\[ K_{\text{sym}}^0 = -3\alpha [3C_2^0 - L_0] + E_F^0(3\alpha - 4) \]
\[ + \left( \frac{2}{3} \frac{m}{m_0} + k - \rho_0 \right) (5 - 3\alpha) \tag{3} \]
\[ Q_{\text{sym}}^0 = 15\alpha [3C_2^0 - L_0] + K_{\text{sym}}^0 (3\alpha - 1) \]
\[ + E_F^0 (2 - 3\alpha). \tag{4} \]

Here, \( \alpha \) is given by,

\[ \alpha = \frac{K_{\text{sym}}^0 + E_F^0 \left( \frac{12}{5} \frac{m}{m_0} - 2 \frac{m}{m_0} \right)}{E_F^0 \left( 3 \frac{m}{m_0} - 2 \frac{m}{m_0} \right) - E_0}. \tag{5} \]

In Eq.(3-5), \( E_F^0 \) is the fermi energy, \( m_0 \) \((\text{m})\) is the effective (free) nucleon mass, \( E_0 \) and \( K_0 \) are energy per particle and nuclear incompressibility of symmetric NM respectively, all of them being defined at \( \rho_0 \). \( k- \) determines the difference between neutron and proton effective masses at \( \rho_0 \).

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Results and discussion

Equation (3) throws a hint that $K_{0}^{0}_{\text{sym}}$ should be linearly correlated with $(3C_{0}^{0} - L_{0})$. In Fig. (1) we plotted $K_{0}^{0}_{\text{sym}}$ as a function of $L_{0}$ (left panel) and $(3C_{0}^{0} - L_{0})$ (right panel) for 500 different mean-field models, both relativistic and non-relativistic, used in the literature. The improvement in the correlation from the left panel to the right panel is realized by looking at the correlation coefficients, as indicated by $r$ in the figure. We have also seen that this correlation between $K_{0}^{0}_{\text{sym}}$ and $(3C_{0}^{0} - L_{0})$ does not depend on the choice of set of models (not shown here) unlike $K_{0}^{0}_{\text{sym}} - L_{0}$ correlation, which shows its universal nature.

Inspired by Eq. (4), we plotted $Q_{0}^{0}_{\text{sym}}$ as a function of $(3C_{0}^{0} - L_{0})$ in the left panel of Fig. (2). However, the correlation is relatively weaker ($r = 0.66$) compared to that of between $K_{0}^{0}_{\text{sym}}$ and $(3C_{0}^{0} - L_{0})$. The primary reason behind that is the error propagation from $K_{0}^{0}_{\text{sym}}$ appearing in the right hand side of Eq. (4). Moreover, different symmetric NM properties ($K_{0}, m_{0}, E_{F}$ and eventually $\alpha$) have quite significant contributions in the value of $Q_{0}^{0}_{\text{sym}}$, which have some variation across concerned 500 mean-field models. If we impart a reasonable limit on $m_{0}$ and $K_{0}$ as $0.65 < m_{0}/\alpha < 0.85$ and $200 \text{ MeV} < K_{0} < 260 \text{ MeV}$ and plot $Q_{0}^{0}_{\text{sym}}$ versus $(3C_{0}^{0} - L_{0})$ for those models which follow these limits, show a very high linear correlation ($r = 0.93$). It again points out the near-universality in the correlations proposed by the simplistic model described above.

**References**