Comparative analysis of various density approximations within the Energy Density Formalism

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Introduction

In low energy heavy-ion reactions, the fusion of two colliding nuclei occurs when they overcome the Coulomb barrier. In other words, there is a possibility of the formation of an excited compound nucleus (*CN*), when the projectile possesses sufficient kinetic energy to penetrate through the Coulomb barrier. It is well known fact that the collision of different nuclei is mainly influenced due to deformation, orientation, excitation energy of projectile, angular momentum etc. In view of this, the study of nuclear dynamics in terms of ion-ion interaction potential has been elaborated using various density approximations within the Energy Density Formalism *(EDF)* [1] using Sly4 Skyrme force parametrization [2].

In *EDF* approach, for composite system, the addition of nucleon densities is introduced generally via two different density approximations, i.e. sudden, and frozen density approximations. In addition to this, the sudden density approximation is modified by the removal of surface thickness factor in central density term and defined as modified sudden density approximation [3]. So, the effect of different barrier characteristics is explored in terms of total interaction potential and corresponding fusion cross-section within extended Wong model [4] for ^{27}Al+^{74}Ge reaction [5]. Furthermore, the impact of quadrupole (β_2) deformed nuclei is analyzed by comparing the results with corresponding spherical target/projectile combinations.

Methodology

Total interaction potential, $V_T(R)$, is mainly expressed as the addition of repulsive potentials (Coulomb (V_C) + centrifugal potential (V_ℓ)) and attractive short-ranged nuclear potential $V_N(R)$. The $V_C(R)$, and $V_\ell(R)$ are well defined in literature. However, the nuclear potential is still not fully understood, due to uncertain nature of interactions involved within the colliding nuclei. The Energy Density Formalism (EDF) [1] forms one of the pertinent way to express the nuclear potential defined as

$$V_N(R) = E(R) - E(\infty)$$

where, $E = \int H(\vec{r})d\vec{r}$ with H as the Hamiltonian density functional, $H(\rho, \tau, J)$, which is function of nuclear density (ρ), kinetic energy density $\tau(\rho)$ and spin-orbit density $J(\rho)$ that are parameterized by the parameters of Skyrme force, Sly4 [2]. Such density-dependent nuclear potential can be differentiated using two different approximations. For composite system, i.e. $\rho = \rho_1 + \rho_2$, $\tau(\rho)$, and $J(\rho)$ are expressed as per prescription used in the following part named as sudden density approximation [1].

$$\tau(\rho) = \tau(\rho_{1n} + \rho_{2n}) + \tau(\rho_{1p} + \rho_{2p}),$$
$$\vec{J}(\rho) = \vec{J}(\rho_{1n} + \rho_{2n}) + \vec{J}(\rho_{1p} + \rho_{2p}).$$

On the other hand, in frozen density approximation [1], the exchange effects in the nucleon densities are excluded, and it reads as

$$\tau(\rho) = \tau(\rho_1) + \tau(\rho_2),$$
$$\vec{J}(\rho) = \vec{J}(\rho_1) + \vec{J}(\rho_2).$$

The fusion cross-sections is calculated by using the ℓ-summed Wong model [4] for spherical nuclei, colliding with $E_{c.m.}$, reads as

$$\sigma(E_{c.m.}) = \frac{\pi}{k^2} \sum_{\ell=0}^{\ell_{max}} (2\ell + 1)P_\ell(E_{c.m.}),$$

here, ℓ_{max} is the maximum value of angular momentum upto which the fusion cross-sections are calculated. When, one uses the interaction between two deformed nuclei of radii

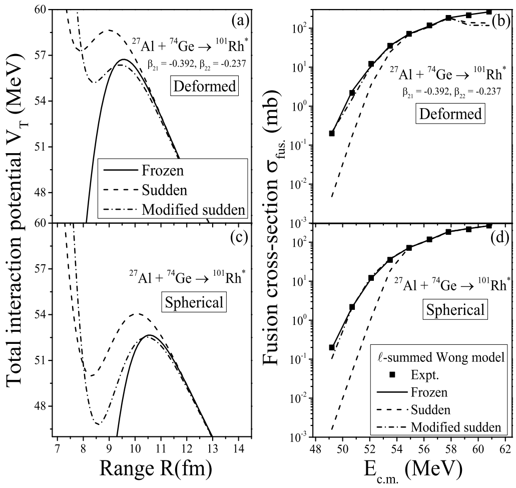

FIG. 1: Comparison of total interaction potentials for (a) deformed and (b) spherical choices of nuclei in ^{27}Al+^{74}Ge reaction. The calculated fusion cross-sections within ℓ-summed Wong model using various density approximations for (c) deformed and (d) spherical nuclei for the same reaction are compared with experimental data [5].

R_i with deformation parameters $\beta_{\lambda i}$ and making orientation angles θ_i with respect to the collision axis, then, the fusion cross-section integrated over the angles θ_i can be obtained as

$$\sigma(E_{c.m.}, \theta_i) = \int_{\theta_i=0}^{\pi/2} \sigma(E_{c.m.}, \theta_i) \sin\theta_i d\theta_i.$$

Results and discussion

In present calculations for ^{27}Al+^{74}Ge [5] reaction, a comparative analysis of interaction potential using various density approximations, i.e. frozen, sudden, and modified sudden density approximations, has been carried out by taking spherical as well as quadrupole deformed (β_2) choices of colliding nuclei. Within Skyrme Energy Density Formalism *(SEDF)*, the interaction potential for two deformed nuclei forming ^{27}Al+^{74}Ge reaction is shown in Fig1(a) for different density approximations. It has been observed from Fig1(a) that the frozen density and modified sudden approximations show a notable difference in the barrier height and barrier position with respect to the potential based on sudden density approximation. It is observed in Fig1(b) that the lower barrier height of frozen density based potential is able to reproduce the experimental data [5] across the barrier within the framework of extended Wong model. However, the fusion cross-sections of the same reaction calculated using modified sudden approach underestimate the data at higher energies, $E_{c.m.}$. In case of sudden density approximation, the fusion cross-sections are underestimated at below as well as above the barrier. The calculations for deformed colliding nuclei are further compared with the spherical choices of projectile and target nuclei in ^{27}Al+^{74}Ge reaction to observe the variation in the total interaction potentials and corresponding fusion cross-sections calculated via extended ℓ-summed Wong model. In Fig1(c), the barrier characteristics associated with different interaction potentials due to spherical nuclei acquire noteworthy deviation of barrier height, and position as compared to that of obtained by deformed ones. Consequently, frozen and modified sudden density approximations are found to address the data nicely across the barrier, as shown in Fig1(d). However, the sudden density approximation seems to suggest fusion hindrance at below barrier region, just like that in case for deformed choice of interaction. In the conclusion, it can be said that for both cases of deformed and spherical nuclei in ^{27}Al+^{74}Ge reaction, frozen and modified sudden density approximations seem to provide favorable choice to address the available data. However, the angular dependence (not shown here) further suggest that frozen approximation is better among the two.

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