Nuclear multi-fragmentation: canonical-grand canonical ensemble transformation

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Among the various existing phenomenological models, statistical models [1] are simple but very suitable to describe the phenomenon of nuclear multi-fragmentation in heavy ion collision at intermediate energies. There are microcanonical, canonical and grand canonical models depending upon the statistical ensemble considered. The calculations are extremely difficult in microcanonical model due to two constraints; though it describes the actual practical senario. Calculations in grand canonical model is easier, hence more commonly used, than canonical model; though the experimental data are usually more close to the canonical model results. So, if one can develop an approximate expression that transforms values for observables from one ensemble to other, value of an observable in one ensemble can be extrapolated from the value calculated in another without direct calculation. In an earlier work[2] such an expression, between canonical and grand canonical ensemble, has been developed for an ideal system of single type of nucleons without dishtinguishing neutorn and proton. In the present work, we have introduced the iso-spin asymetry, thus extended it to the case of real nuclei. This will be useful in cases of temperature measurement by double isotope ratio method, symmetry energy from isoscaling and isobaric yield ratio parameters.

In statistical model [1], we consider an excited system of Z_0 protons and N_0 neutrons, has expanded to a higher volume greater than normal nuclear volume producing fragments. It is assumed that this system attains equilibrium(thermmal and chemical) at freeze-out condition when the temperature is T and volume is V_f. In canonical model each system in ensemble has exactly the same number of particles whereas in grand canonical model particle number can vary from zero to infinity but the average should be $\langle Z_0\rangle_{f_n,fz} = Z_0$ and $\langle N_0\rangle_{f_n,fz} = N_0$, where f_z, f_n are the proton and neutron fugacities. It is well known that the partition function in grand canonical ensemble can be written in terms of sum of the partition functions in canonical ensembles of different size as,

$$Q_{f_n,f_z} = \sum_{N_0,Z_0=0}^{\infty} Q_{N_0,Z_0} \,.\, e^{f_z Z_0 + f_n N_0}$$

where Q_{N_0,Z_0} and Q_{f_n,f_z} are the partition functions in canonical and grand canonical ensembles respectively. Probability of each canonical source in grand canonical ensemble is given by,

$$P_{f_n,f_z}(N_0, Z_0) = \frac{Q_{N_0,Z_0} \,.\, e^{f_z Z_0 + f_n N_0}}{Q_{f_n,f_z}}$$

Now, we consider any observable R. The canonical average value of this observable is $R_c(N_0, Z_0)$ where the grand canonical average value is $R_{gc}(f_n, f_z)$ and it can be written as,

$$R_{gc}(f_n, f_z) = \sum_{N_0,Z_0=0}^{\infty} R_c(N_0, Z_0) \,.\, P_{f_n,f_z}(N_0, Z_0)$$

Then we expand $R_c(N_0, Z_0)$ in Taylor series around $(\langle N_0\rangle_{f_n,fz}, \langle Z_0\rangle_{f_n,fz})$ and taking up to 2nd order term and following some steps

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finally we get [3],

$$R_c(\langle N_0\rangle_{f_n,f_z},\langle Z_0\rangle_{f_n,f_z}) \approx R_{gc}(f_n,f_z) - \frac{1}{2}\sigma_n^2 \frac{\partial^2 R_{gc}}{\partial \langle N_0\rangle^2}\bigg|_{\langle N_0\rangle_{f_n,f_z},\langle Z_0\rangle_{f_n,f_z}} - \frac{1}{2}\sigma_z^2 \frac{\partial^2 R_{gc}}{\partial \langle Z_0\rangle^2}\bigg|_{\langle N_0\rangle_{f_n,f_z},\langle Z_0\rangle_{f_n,f_z}} - \sigma_{nz}\frac{\partial^2 R_{gc}}{\partial \langle N_0\rangle \partial \langle Z_0\rangle}\bigg|_{\langle N_0\rangle_{f_n,f_z},\langle Z_0\rangle_{f_n,f_z}}$$

where σ_n, σ_z, σ_{nz} are the particle number fluctuations in grand canonical ensemble defined as,

$$\sigma_n^2 = \sum (N_0 - \langle N_0\rangle_{f_n,f_z})^2 P_{f_n,f_z}$$

$$\sigma_z^2 = \sum (Z_0 - \langle Z_0\rangle_{f_n,f_z})^2 P_{f_n,f_z}$$

$$\sigma_{nz} = \sum (N_0 - \langle N_0\rangle_{f_n,f_z})(Z_0 - \langle Z_0\rangle_{f_n,f_z}) P_{f_n,f_z}$$

In arriving at the final step, we have approximated that the particle number fluctuations are negligible.

In right-hand side all the terms are to be calculated using grand canonical ensemble. So using this equation, starting from grand canonical calculation we can extrapolate corresponding canonical results without direct calculations.

Since in our case, both canonical and grand canonical models are analytically solvable, we can check the validity and accuracy of this transformation in the context of nuclear multifragmentation. We have considered a system of $Z_0 = 28$ protons and $N_0 = 30$ neutrons at freeze-out volume $V_f = 3V_0$ and studied at two different temperatures. We have taken the observables, such as total multiplicity and largest cluster size and calculated the values from canonical model, grand canonical model and using transformation relation, then we have compared the results. We have also studied multiplicities of each and every individual fragments and ploted mass distribution[FIG. 1]. In each case we see that the transformation relation gives very accurate results.

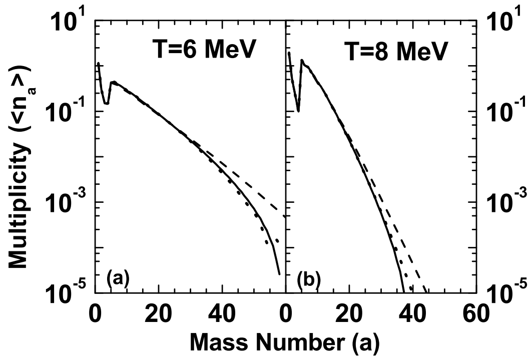

FIG. 1: Mass distribution of fragments produced from fragmentation of a source of 28 proton and 30 neutron, calculated using canonical(dotted line), grand canonical(dashed line)model and transformation relation(solid line) for two different temperatures, T=6MeV(left) and T=8MeV(right).

TABLE I: The grand canonical values and the values of canonical extrapolation are compered with exact canonical values for different observables obtained from the fragmentation of the source of mass 58, charge 28 at $V_f = 3V_0$ and two temperatures, T=6MeV and T=8MeV.

Observables	T(MeV)	R_{gc}	R_c	$R_{gc\,to\,c}$
$\langle n\rangle_{tot}$	6	5.994	6.155	6.116
	8	9.131	9.184	9.171
$\langle Z_{max}\rangle$	6	10.293	10.752	10.724
	8	6.653	6.796	6.798

There are some limitations of this approximate expresstion. In those conditions, where particle number fluctuations can not be neglected or when the value of the observable itself is very small, so that the higher order terms become significant, this relation is not very accurate.

References

[1] C.B. Das, S. Das Gupta, W.G. Lynch, A.Z. Mekjian, M.B. Tsang, Phys. Rep. **406**, 1 (2005).

[2] G. Chaudhuri, F. Gulminelli, and S. Mallik, Phys. Lett. B **724**, 115 (2013).

[3] P. Das, S. Mallik, and G. Chaudhuri, Accepted in Phys. Rev. C (2017).