Study of neutron scattering in the nonlocal framework

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Introduction

The nucleon-nucleus interaction is known to be nonlocal in nature [1, 2] and has been extensively studied since 1950's. Incorporation of the nonlocal effects leads to the Schrödinger equation in the integro-differential form given by

$$\left[\frac{\hbar^2}{2\mu}\nabla^2 + E\right]\Psi(\mathbf{r}) = -\left[V_{so}\mathbf{L}\cdot\boldsymbol{\sigma}\right]\Psi(\mathbf{r}) + \int V(\mathbf{r},\mathbf{r}')\Psi(\mathbf{r}')d\mathbf{r}' , \quad (1)$$

where μ is the reduced mass of the nucleon-nucleus system, E is the center of mass energy, $V_{so}\,\mathbf{L}\cdot\boldsymbol{\sigma}$ is the local spin-orbit interaction, $V(\mathbf{r},\mathbf{r}')$ is the nonlocal interaction and $\Psi(\mathbf{r})$ is the scattering wave function.

A factorized form for the nonlocal interaction was proposed by Frahn and Lemmer [2], written as

$$V(\mathbf{r},\mathbf{r}') = \frac{e^{-\left|\mathbf{r}-\mathbf{r}'\right|^2/\beta^2}}{\pi^{3/2}\beta^3}\, U\left(\frac{|\mathbf{r}+\mathbf{r}'|}{2}\right) \quad (2)$$

where β is the nonlocal range parameter and U is the energy and mass independent interaction.

This type of potential was first investigated by Perey and Buck in 1960 [3]. The nonlocal optical potential parameters were obtained for neutron scattering off nucleus by fitting the neutron elastic scattering data on ^{208}Pb at 7 MeV and 14.5 MeV. Recently, a new set of nonlocal optical potential parameters (TPM15) for proton and neutron scattering off nuclei has been obtained by fitting the nucleon scattering data on nuclei ranging from ^{27}Al to ^{208}Pb with incident energies around 10 MeV to 30 MeV [4].

A readily implementable technique using the second mean value theorem (MVT) of the integral calculus to solve integro-differential equation was developed by present authors in Ref.[5]. In the present work we calculate differential cross sections for neutron scattering of ^{56}Fe and ^{100}Mo within this new technique using TPM15 parameters [4] in the energy range smaller than or equal to 10 MeV.

Solving the integro-differential equation

To solve Eq.(1), we start with the partial-wave decomposed integro-differential equation written as

$$\left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + \frac{2\mu\, E_{\rm cm}}{\hbar^2}\right] u_{jl}(r) = \frac{2\mu}{\hbar^2}\left[-U_{so}(r)u_{jl}(r) + \int_0^\infty g_l(r,r')u_{jl}(r')\,dr'\right]. \quad (3)$$

The nonlocal kernel is given as

$$g_l(r,r') = \left(\frac{2rr'e^{(-r^2-r'^2)/\beta^2}}{\sqrt{\pi}\,\beta^3}\right) \times \int_{-1}^{1} U\left(\frac{|\mathbf{r}+\mathbf{r}'|}{2}\right) e^{2rr'\cos\theta/\beta^2} P_l(\cos\theta)\, d(\cos\theta) \quad (4)$$

with θ being the angle between the vectors $\mathbf{r}$ and $\mathbf{r}'$. The forms and parameters used for $U(|\mathbf{r}+\mathbf{r}'|/2)$, $U_{so}(r)$ and β are those given in Ref.[4].

Application of MVT [5] converts the non-homogeneous equation to a homogeneous equation of the form

$$\left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + \frac{2\mu E_{\rm cm}}{\hbar^2}\right] u_{jl}(r) = \frac{2\mu}{\hbar^2}\left[-U_{so}(r) + U_l^{\rm eff}(r)\right] u_{jl}(r) , \quad (5)$$

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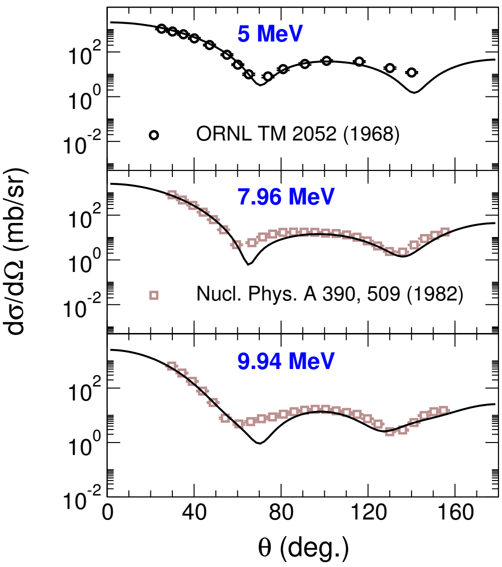

FIG. 1: Calculated angular distributions for neutron scattering off ^{56}Fe along with the data. Black solid line represents theoretically calculated results.

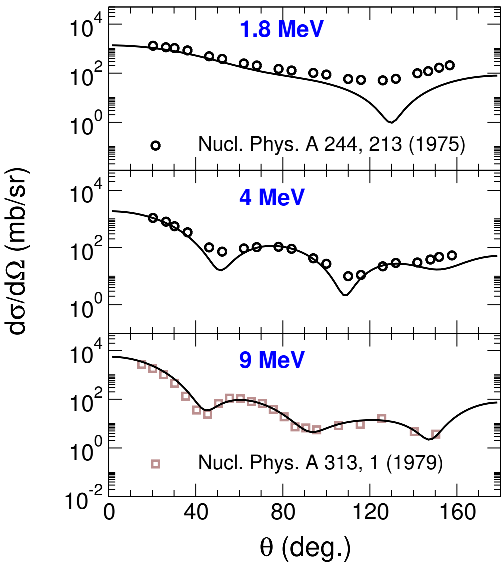

FIG. 2: Calculated angular distributions for neutron scattering off ^{100}Mo along with the data. Black solid line represents theoretically calculated results.

where $U_l^{\rm eff}(r) = \int_0^\infty g_l(r,r')\,dr'$. For improving accuracy of the results, an iterative scheme has been proposed which is initiated by the above homogeneous equation.

Results and Conclusions

To test the overall accuracy of the calculated observables, we compare them with the data. As representative cases, in Figs.1-2 we show the calculated angular distributions for ^{56}Fe and ^{100}Mo represented by black solid line along with the data. The computed results are in very good agreement with the data over the entire range of energies considered here.

Hence, it can be concluded that the new method proposed by present authors in Ref.[5] is robust and independent of the exact shape of the kernel, thus being versatile. It has been validated by successfully studying neutron scattering off ^{56}Fe and ^{100}Mo targets in the energy range 0.1-10 MeV.

Acknowledgments

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