Scattering from a semi-infinite non-Hermitian potential

Dona Ghosh [1*] and Zafar Ahmed [2]
[1] *Department of mathematics, Jadavpur University, Kolkata-700032, India and*
[2] *Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai - 400085, India*
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Nucleus-nucleus scattering is generally described by non-Hermitian complex potential. The absorptive (imaginary) part of the potential is supposed to represent non-elastic channels which reduces the elastically scattered flux. The relevant model called the optical model, is found to be phenomenologically suitable for interpreting the elastic scattering. In the optical model, the Hamiltonian is non-hermitian and hence the condition for unitarity of the $S(E)$ matrix ($|S(E)| = 1$), is replaced by the condition for pseudo-unitarity ($|S(E)| < 1$). Recently, novel interesting model are of the form $V_c(x) = V_r(x) + iV_i(x)$, where $V_i(x)$ is not essentially absorbing [1-5].

For a non-Hermitian complex potential of the above form which vanishes at $\pm\infty$ and which is spatially non-symmetric, the reflection amplitudes display non-reciprocity ($r_L \neq r_R$) whereas transmission amplitudes are reciprocal $t_L = t_R$ [1]. For non-hermitian scattering potential there may exist a special real energy (E_*) named as Spectral singularity [2], where all three probabilities (transmission, reflection from left and reflection from right) $T(E), R_L(E)$ and $R_R(E)$ become infinity. Very interesting exactly solvable models are available [3] where one gets explicit expression of E_* and explicit parametric conditions on the non-Hermitian potential. These two important observations give rise to a new experimentation where coherent (identical) beams are injected into a non-Hermitian optical medium from left and right. In the coherent scattering the two port $S(k)$ matrix is given as $|S(k)| = |r_L(k)r_R(k) - t^2(k)|$ [2,4]. It has been further claimed that if spectral singularity occurs at $E_* = k^2$, $|S(-k)|$ becomes zero at $E = E_*$ signifying perfect absorption of coherent beams [4] in the non-Hermitian medium. Very interesting exactly solvable models of Coherent Perfect Absorption (CPA) have been proposed [5].

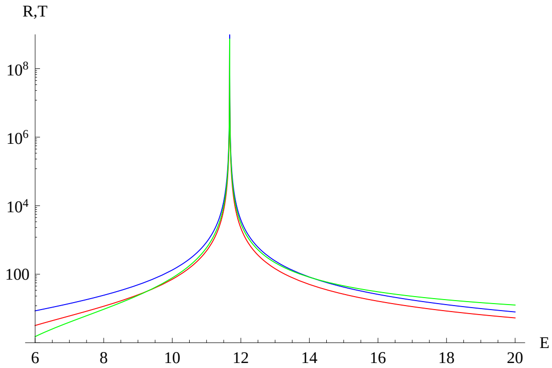

FIG. 1: Reflection coefficient from left (blue line) and right (red line) and Transmission probability for left or right (green line) as a function of E. Spectral singularity has been observed at E_*=11.6736

In a scattering process $V_i(x)$ needs to be vanishing asymptotically, but V_r(x) may also be semi-infinite. By semi-infinite we mean that $V_r(x < 0) = 0$ and $V_r(x > 0) = V_1$. In this interesting situation we find that both r and t are non-reciprocal, specially $t_R = (k_R/k_L)t_L$. Here $k_L = \sqrt{2\mu E}/\hbar, k_R = \sqrt{2\mu(E - V_1)}/\hbar$. Nevertheless, the transmission probability $T(E)$ remains reciprocal again.

The crucial question arising here is as to whether a semi-infinite $V_r(x)$can also give rise to CPA with a modified two-port S-matrix with $|S(k)| = |r_L r_R - t_L t_R|$. For this we pro-

*Electronic address: rimidonaghosh@gmail.com

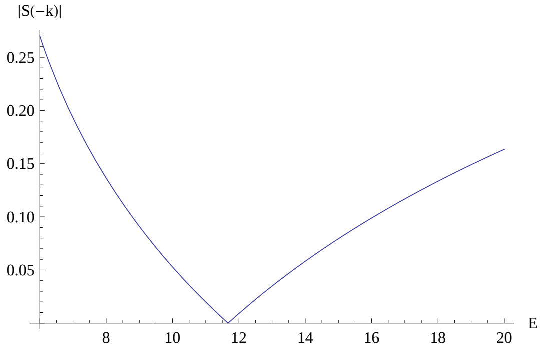

FIG. 2: Variation of determinant of two port S matrix $S(-k)$ with energy. $|S(-k)|$ becomes zero for the same E_*=11.6736 signifying coherent perfect absorption of coherent beams.

pose the first semi-infinite model as

$$V_c(x) = V_1\Theta(x) + iV_2\delta(x) \quad (1)$$

where $\Theta(x<0)=0, \Theta(x>0)=1$. One can solve the Schrodinger equation when scattering occurs from left and obtain the solution as

$$\psi(x<0) = A\exp(ik_Lx) + B\exp(-ik_Lx) \quad (2)$$

$$\psi(x>0) = C\exp(ik_Rx) \quad (3)$$

We demand continuity of $\psi(x)$ at $x=0$ and discontinuity of first derivative of $\psi(x)$ at $x=0$, due to presence of Dirac Delta function as

$$\frac{d\psi_>(x)}{dx}\mid_{x=0} - \frac{d\psi_<(x)}{dx}\mid_{x=0} = \frac{2\mu iV_2}{\hbar^2}\psi_<(0). \quad (4)$$

Using the above equations we obtain the reflection amplitude (r_L) and transmission amplitude (t_L) as

$$r_L = B/A = \frac{k_L - k_R + \alpha}{k_L + k_R - \alpha}, \quad (5)$$

$$t_L = C/A = \frac{2k_L}{k_L + k_R - \alpha} \quad (6)$$

(where $\alpha = \frac{2\mu V_2}{\hbar^2}$). Now similar exercise can be done for scattering from right and one can obtain the solutions as

$$\psi(x>0) = A'\exp(-ik_Rx) + B'\exp(ik_Rx) \quad (7)$$

$$\psi(x<0) = C'\exp(-ik_Lx) \quad (8)$$

Now using equations (7) and (8) we obtain the reflection amplitude from right(r_R) and transmission amplitude from right (t_R) as

$$r_R = B'/A' = \frac{k_R - k_L + \alpha}{k_R + k_L - \alpha} \quad (9)$$

$$t_R = C'/A' = \frac{2k_R}{k_R + k_L - \alpha} \quad (10)$$

Eventually, we get the zero of $|S(-k)|$ and the spectral singularity at a real energy

$$E_* = \left(\frac{\beta^2 + V_1}{2\beta}\right)^2, \quad \beta = \frac{\sqrt{2\mu}}{\hbar}V_2. \quad (11)$$

Taking $2\mu = 1 = \hbar^2$, and $V_1 = 5, V_2 = 6$, we carry out calculations for $T(E), R_L(E)(= |r_L(E)|^2)$ and $R_R(E)(= |r_R(E)|^2)$ which has been shown in Fig.1 by blue, red and green lines respectively. One can observe that $T(E), R_L(E)$ and $R_R(E)$ become simultaneously infinity at a special energy E_*=11.6736 displaying spectral singularity at E_*=11.6736. Interestingly, we also find that the determinant of two port S-matrix: $|S(-k)|$ becomes simultaneously zero at the same $E = E_*$. (see Fig. 2). So we confirm the occurrence of coherent perfect absorption for this simple semi-infinite non-Hermitian Hamiltonian. For more interesting and involved models, the work is in progress.

References

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