Tracking Radial independence of half lives and a radial distance formula for amicable use of Viola-Seaborg relation

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1. Introduction

Nuclear radial distance is a prerequisite for generating any alpha decay half-life formula by taking a suitable effective potential. We study the emission process of alpha particles from an isolated quasi-bound state generated by an effective potential to a scattering state. The effective potential is expressed in terms of Frahn form of potential which is exactly solvable and an analytical expression for decay width is obtained in terms of wavelength and potential. We then derive a closed form expression for the decay half life in terms of the parameters of the potential, Q-value of the system, mass and proton numbers of the nuclei. From the nature of variations of half life as a function of radial distance, we trace the radial independence region where decay time is almost constant. Finally by overviewing our results, we present a radial distance formula valid for light, heavy and superheavy nuclei.

TABLE I: Q-values in MeV, $log_{10}T^{expt}_{1/2}$=T^{ex}, $log_{10}T^{c}_{1/2}$=T^{c}, $log_{10}T^{f}_{1/2}$=T^{f}, radial distance R^f, range of radial independence = R^w for $l = 0$ of different systems .

Nu.	Q	T^{ex}(s)	T^{c}(s)	T^{f}(s)	R^f(fm)	R^w(fm)
$^{106}_{52}$	4.290	-4.09	-4.5609	-4.5602	9.2	8.9-13.0
$^{114}_{56}$	3.534	1.77	1.5650	1.5657	9.4	8.9-11.5
$^{218}_{84}$	6.115	2.26	2.2292	2.2299	10.5	9.6-12.1
$^{216}_{90}$	8.071	-1.58	-1.5837	-1.5830	10.5	10.1-11.3
$^{232}_{94}$	6.716	3.30	3.4097	3.4105	10.6	10.2-12.2
$^{250}_{100}$	7.556	3.26	3.1613	3.1613	10.8	10.4-11.3
$^{290}_{116}$	10.990	-2.09	-2.3032	-2.3808	11.0	10.6-11.4

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2. Formualtion

By considering the Frahn potential[1] as the effective potential V_{eff}, point charge coulomb potential V_c^P, Coulomb wave function $F_l(r)$ and wave function $u_{nl}(r)$, we write the expression for decay width[2] as;

$$\Gamma = \frac{4\mu}{\hbar^2 k} \; \frac{|\int_0^R F_l[V_{eff}(r) - V_C^P(r)]u_{nl}(r)dr|^2}{\int_0^R |u_{nl}(r)|^2 dr}. \quad (1)$$

The α-decay half life is related to the decay width by $T_{1/2} = \frac{\hbar ln2}{\Gamma}$. Using the Gamma value (1) $T_{1/2}$ is expressed as

$$T^c_{1/2} = \frac{0.693\hbar^3 k}{4\mu} \; \frac{1}{J}, \quad (2)$$

$\mu = m_n \frac{A_\alpha A_D}{A_\alpha + A_D}$ is the reduced mass of the system with m_n giving the mass of a nucleon. A_α represent the mass number of α particle, A_D represent the mass number of the daughter nucleus.

$$J = |\int_0^R F_l[V_{eff}(r) - V_C^P(r)]u_{nl}(r)dr|^2. \quad (3)$$

We are taking into account the problem of α+nucleus system having a specific energy value i.e. Q-value, Sommerfeld parameter η and parameter $\rho = kR$ such that $\eta\rho \leq 50$ and $\rho \approx 10$. Thus, in this context, we now use the power series expansion and write the Coulomb wave function $F_l(r)$ as

$$F_l^{ps}(r) = C_l \rho^{l+1} G_l, \quad (4)$$

$$(n+1)(n+2l+2)G_{n+1} = 2\eta\rho G_n - \rho^2 G_{n-1}, \quad (5)$$

$$G_0 = 1, G_1 = \frac{\eta\rho}{(l+1)}, G_l = \sum_{j=1}^{500} G_j, \quad (6)$$

$$C_l^2 = \frac{P_l(\eta)}{2\eta}\frac{C_0^2(\eta)}{(2l+1)}, \quad (7)$$

$$P_l(\eta) = \frac{2\eta(1+\eta^2)(4+\eta^2)\ldots(l^2+\eta^2)2^{2l}}{(2l+1)[(2l)!]^2}. \quad (8)$$

We find that

$$F_l(r) = x_m F_l^{ps}(r), x_m = 70. \quad (9)$$

The integral J now changes to $J = |c_f F_l(R)|^2 = |c_f x_m F_l^{ps}(R)|^2$. For a typical α+nucleus system, the value of c_f can be written as

$$c_f = \frac{\sqrt{J}}{|x_m F_l^{ps}(R)|}. \quad (10)$$

We plot the $log_{10}T^c_{1/2}$ vs r and find the radial independence region R^w and trace a particular radial distance R^f in the radial independence region where the decay time is remaining constant. Thus by analyzing the constancy region from the plots of $log_{10}T^c_{1/2}$ for various systems we zero in to a radial distance formula which is given by;

$$R^f = r_0(A_\alpha^{1/3} + A_D^{1/3}) + 2.72 + \delta, \quad (11)$$

where $r_0 = 0.97$ fm and $\delta = 0.5$. Using the above found J value and R^f, the logarithmic half-life $T_{1/2}$(updated Viola Seaborg relation) becomes

$$logT^f_{1/2} = a\chi + c + d + b_l, \quad (12)$$

$$a = 0.96, \chi = Z_\alpha Z_D\sqrt{\frac{A_\alpha A_D}{(A_\alpha + A_D)Q_\alpha}}, \quad (13)$$

$$c = -2logS, d = -45.26, b_l = log(q_l), \quad (14)$$

$$q_l = \frac{2\eta(2l+1)}{P_l(\eta)\rho^{2l}}, S = c_f x_m R^f G_l \frac{A_\alpha A_D\sqrt{Z_\alpha Z_D}}{A_\alpha + A_D}. \quad (15)$$

3. Results

Fig.1 shows the half life T^c, effective potential V(r) and wave function u_{nl} as a function of r(fm) plots where the radial independence region R^w for Rn^{220}_{86} nucleus comes out to be 8.9-13.0 fm with R^f= 10.5 fm . The R^f when used in eqn(12) gives T^f=1.9116 s which matches closely with the experimental result of 1.74 s. T^c, T^f, R^f and R^w values for different systems have been enlisted in Table 1 which show good agreement with experimental results. Hence, with the radial distance R^f values, Q-values and mass and proton numbers with us we can find the decay half life for any nuclei.

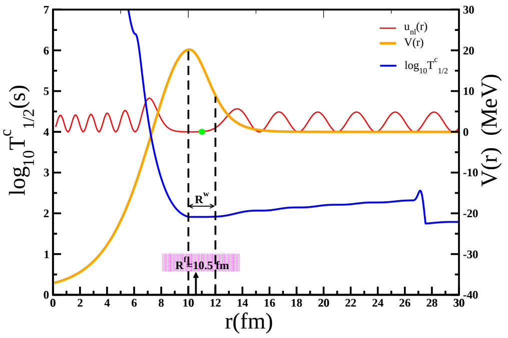

FIG. 1: Plot of the $log_{10}T^c_{1/2}$, effective potential $V_{eff}(r)$ and wave function u_{nl} as a function of r(fm) for Rn^{220}_{86} nucleus. Green dot represents the moving particle through the barrier.

4. Conclusion

Finally, we affirm that for any system taken into consideration, a radial independence region can be tracked to be used amicably in the updated Viola- Seaborg relation for prediction of α decay half lives.

Acknowledgments

We gratefully acknowledge the computing and library facilities extended by the Institute of Physics, Bhubaneswar.

References

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