Study of the effect of the slope parameter on the r-mode instability using Simple Effective Interaction

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Introduction

Neutron stars (NSs) may go through a number of instabilities. The important one of these is the r-mode instability. The r-mode is a toroidal mode of oscillation whose restoring force is the Coriolis force. It has received a lot of attention as this oscillation mode leads to the emission of gravitational waves (GW) in NSs. Also it proposes an explanation for the sub-breakup spin rates of low mass x-ray binaries (LMXBs), millisecond radio pulsars (MSRPs) and young, hot neutron stars [1-5]. The physics of these oscillations is important in understanding the relation between the microscopic properties of dense matter (such as its viscosity and neutrino emissivity) and the macroscopic, observable properties of NSs (such as their spin frequency, temperature (T), mass (M), and radius (R)). If these waves could be potentially detectable, then it could provide invaluable information on the internal structure of the star and constraints on the nuclear equation of state (EoS), particularly to restrict the density dependence of the symmetry energy, $E_{sym}(\rho)$. The later one is one of the very important ingredients of the nuclear EoS required for understanding various properties of isospin rich nuclei and NSs. Though the value of the symmetry energy at saturation density ρ_0 is comparatively well established (arround 30MeV) and its behaviour below ρ_0 is rather better understood [6], it is not well constrained above ρ_0. The predictions of $E_{sym}(\rho)$ from different approaches strongly diverge above ρ_0. The study of r-modes can be used to extract information on $E_{sym}(\rho)$ complementary to the one obtained from various methods[7]. In this work, the effect of the symmetry energy slope parameter L on the r-mode instability is studied using simple finite range effective interaction (SEI).

Formalism

The amplitude of r-modes evolves with a time dependence $e^{i\omega t - t/\tau}$ as a consequence of ordinary hydrodynamics and the influence of the various dissipative processes, where, ω is the frequency of the r-mode. The imaginary part of $(1/\tau)$ is determined by the combined dissipative effects of gravitational radiation, viscosity, etc. The dissipative time scale of an r-mode is given by [3,4],

$$\frac{1}{\tau_i} = -\frac{1}{E}\left(\frac{dE}{dt}\right)_i , \quad (1)$$

where, the index i refers to the various dissipation mechanisms and in the present case the gravitational wave radiation (GR) and the shear viscosity (SV). The effect of bulk viscosity is ignored here as it is not important for $T < 10^9$ K. In Eq. (2), E is the energy of the mode, and $(dE/dt)_i$ is the rate of dissipation associated with each mechanism. In the limit of small angular velocity E is given by [3],

$$E = \frac{1}{2}\alpha^2\,\Omega^2 R^{-2l+2}\int_0^R \rho(r)\, r^{2l+2}\, dr , \quad (2)$$

where, Ω is the angular velocity of the unperturbed star, amplitude, α, of the r-mode is taken as an arbitrary constant and ρ is the mass density of the star. The critical angular velocity, Ω_c, above which the r-mode is unstable, is obtained from the condition $\frac{1}{\tau} = 0$ (i.e. $\tau_{GR} = -\tau_{SV}$) and is given for l=m=2 [3],

$$\Omega_c = \Omega_0 \left(\frac{-\tilde{\tau}_{GR}}{\tilde{\tau}_{SV}}\right)^{2/11} \left(\frac{10^8\ K}{T}\right)^{2/11} \quad , \qquad (3)$$

where, $\Omega_0 = \sqrt{\pi G \bar{\rho}}$, $\bar{\rho}(= 3M/4\pi R^3)$ is the mean density of the star, $\tilde{\tau}_{GR}\left(= \tau_{GR}\left(\frac{\Omega}{\Omega_0}\right)^{2l+2}\right)$ is the fiducial gravitational-radiation time scale and $\tilde{\tau}_{SV}\left(= \tau_{SV}\left(\frac{\Omega}{\Omega_0}\right)^{1/2}\left(\frac{T}{10^8\ K}\right)\right)$ is the fiducial viscous time scale. The present calculation of critical frequency for the r-mode, given in Eq.(3), is done using the finite range simple effective interaction (SEI) [8 and references therein]

$$v_{eff} = t_0(1 + x_0 P_\sigma)\delta(r) \quad + \frac{t_3}{6}(1 + x_3 P_\sigma) \left(\frac{\rho}{1+b\rho}\right)^{\gamma} \delta(r) + (W + BP_\sigma - HP_\tau - MP_\sigma P_\tau)f(r) \,. \quad (4)$$

Results and Discussion

In order to calculate the critical frequency, the NS properties, such as, mass density as a function of distance from the centre, radius of NS, crust-core transition density and pressure are calculated for different L values of the EOS, \gamma=1/2, of SEI that corresponds to the incompressibility value 240 MeV. The slope of the symmetry energy L is defined as $L=3\rho_0 (dE_s(\rho)/d\rho)|_{\rho=\rho_0}$, E_s being the symmetry energy. With these properties of the NS used as input, the fiducial time scales are calculated for the EoSs corresponding to different L values for NSs of masses 1.4 $M_\odot$ and 1.8 $M_\odot$ and the results are which is given in Table 1.

The critical frequency as a function of temperature is now calculated using these fiducial time scales and the results are shown in Figure 1. In the same figure the measured frequencies [5] are also shown. It is found that the majority of the star lies outside the stability window predicted by the SEI except 4U-1608-52 and the 4U-1636-56. Our results are qualitatively similar to the ones obtained in Ref. [9,10]. In both the works the viscous dissipation at the boundary layer between crust and core, is explicitly considered. It is obtained that the r-mode instability region is smaller for larger values of L. These results are similar to the finding of Vidaña et al. [11]. It is also found that the instability window drops by 18-27 Hz for critical temperature in the range (1-10) $\times 10^8$ K when the mass of a neutron star is raised from 1.4 $M_\odot$ to 1.8 $M_\odot$ as can be seen from figure 1.

1.4 $M_\odot$		
L (in MeV)	$\tilde{\tau}_{GR}$ (in s)	$\tilde{\tau}_{SV}$ (in s)
70.77	-2.6756	31.1579
85.00	-3.3792	35.0856
100.00	-4.1176	40.7640
1.8 $M_\odot$		
L (in MeV)	$\tilde{\tau}_{GR}$ (in s)	$\tilde{\tau}_{GR}$ (in s)
70.77	-0.6001	34.7749
85.00	-0.8166	38.9509
100.00	-1.0229	45.2790

Table 1: The fiducial time scales of neutron star

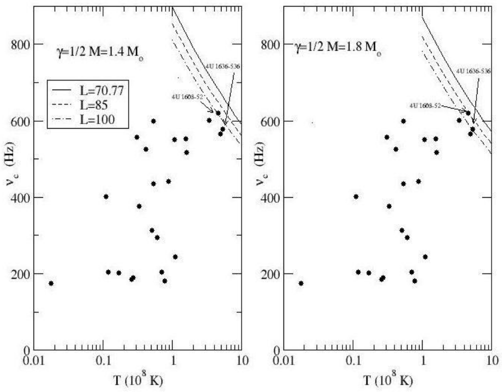

Figure 1: The temperature dependence of critical frequency (ν_c) for a neutron star with mass 1.4 $M_\odot$ (right) and 1.8 $M_\odot$ (left).

References

[1] N. Anderson, Astrophy. J. **502**, 708 (1998).
[2] J. L. Friedman and S. M. Morsink, Astrophy. J. **502**, 714 (1998).
[3] L. Lindblom, B. J. Owen, and S M. Morsink, Phys. Rev. Lett. **80**, 22 (1998).
[4] L. Lindblom, B. J. Owen, and G. Ushomirsky, Phys. Rev. D **62**, 084030 (2000).
[5]B. Haskell, N. Degenaar, andW. C. G. Ho, Mon. Not. R. Astron. Soc. **424**, 93 (2012).
[6] M.B. Tsang etal., Prog. Part. Nucl. Phys. **66**, 400 (2011)
[7] S. Mahmoodifar and T. Strohmayer, Astrophy. J. 773, 140 (2013).
[8] B Behera, T R Routray and S K Tripathy,J. Phys. G: Nucl. Part. Phys. 36 (2009)
[9] Ch. C. Moustakidis, Phys. Rev. C **91**, 035804 (2015).
[10] D. H.Wen,W. G. Newton, and B.A. Li, Phys. Rev. C **85**, 025801 (2012).
[11] I. Vidaña, Phys. Rev. C **85**, 045808 (2012).